

1. Is the sequence $\{1, 4, 16, 64, 256\}$ an arithmetic sequence, a geometric sequence, or neither type of sequence? For your answer, write the whole word **arithmetic**, **geometric**, or **neither type**, whichever is correct.

2. Let $A = \left\{-\frac{1}{4}, \frac{2}{3}, -2, 17\right\}$. If one of the four members of A is selected at random and substituted for x , find the probability that $x > x^3$. Express your answer as a common fraction reduced to lowest terms.

3. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always**, **Sometimes**, or **Never**—whichever is correct.

$$|3x| = 3x$$

4. The focus of the parabola whose equation is $9x^2 - 24x + 72y + 16 = 0$ is $\left(\frac{4}{3}, k\right)$. Find the value of k .
5. Let k represent a positive integer. If the roots of $x^2 + 10x + k = 0$ are non-real, find the smallest possible value of k .
6. Let k , w , and p each represent positive integers such that $k > 2$, $w > 2$, and $p > 2$. Each of k cases contains w boxes, and each box contains p “items.” If $w \neq p$ and the total number of “items” in the k cases is 90, find the smallest possible value of $(w + p)$.
7. When 2^{2010} is written as an integer, how many digits are there in the integer?

8. If the speed of the current remains constant at 5.5 mph, a boat can travel 246 miles downstream in the same time it can travel 180 miles upstream. Find the number of miles per hour in the speed of the boat in still water. Express your answer as a **decimal**.
9. By substituting 1, 2, 3, 4, 5, and 6 for x , in that order, in a polynomial function in x , the first six terms are respectively: 22, 57, 132, 268, 489, and 822. If $P(x)$ is the polynomial function of lowest degree satisfying the given, find $P(25)$.
10. Find the smallest positive integer that has exactly 44 distinct positive divisors.
11. If $x^4 - 1 = 624$, then find the largest possible value of $x^3 + 2x^2 - 3x + 7$.
12. The sum of the squares of the roots for x of $3x^3 + 81x^2 + kx - 287 = 0$ is 785. Find the value of k .
13. Find the value of x such that $\log_2(x+3) - \frac{1}{\log_5(2)} + \log_2(3) = 4$. Express your answer as an improper fraction reduced to lowest terms.
14. Let x and n represent two-digit positive integers for which the tens digit is non-zero and for which $n > x$. Find the value of x if the sum of the consecutive positive integers from 1 through $(x-1)$ is the same as the sum of the consecutive positive integers from $(x+1)$ through n .

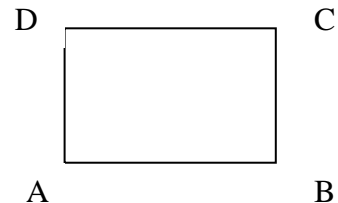
15. Find the degree of the following polynomial: $3x(x+5) + 6x^2(x^2-7)^3 + 44x - 179$

16. A quadratic function $f(x) = ax^2 + bx + c$ has zeroes of 3 and 5. The graph of $f(x)$ passes through the point $(2, -9)$. If a , b , and c represent integers, find the largest possible value of $(a + b + c)$.

17. The eleven positive integers from 1 through 11 are placed in a hat and drawn out at random one at a time without replacement. Find the probability that the odd integers were drawn out as the first six draws. Express your answer as a common fraction reduced to lowest terms.

18. If $x = 1 + \frac{3}{1 + \frac{3}{1 + \dots}}$, then $x = \frac{k + \sqrt{w}}{p}$, where k , w , and p are positive integers. Find the value of $(k + w + p)$

19. In the diagram, $ABCD$ is a rectangle with $AB = 45$ and $BC = k$. Let $i = \sqrt{-1}$. If $|45 + ki| = 53$, find the perimeter of the rectangle.



20. A circle has the line $x - 2y + 4 = 0$ tangent at the point $(0, 2)$ and $y = 2x - 7$ tangent at $(3, -1)$. Find the exact area of this circle.

2009 RA

Algebra II

Name ANSWERS

School _____

(Use full school name – no abbreviations)

_____ Correct X 2 pts. ea. =

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. Geometric (Must be this word.)11. 1672. $\frac{1}{2}$ (Must be this reduced common fraction.)12. -843. Sometimes (Must be this word.)13. $\frac{71}{3}$ (Must be this reduced improper fraction.)4. -214. 355. 2615. 86. 816. -247. 60617. $\frac{1}{462}$ (Must be this reduced common fraction.)8. 35.5 (Must be this decimal, "mph" optional.)18. 169. 8628419. 14610. 1536020. 5π (Must be this exact answer.)