

Oral Competition
Division A - 2009 ICTM Regional Competition
Voting Theory Questions

1. In the 2002 Illinois Gubernatorial Democratic Primary the election results were as follows:

Candidate	Popular Vote
Rod Blagojevich	457,197
Paul Vallas	431,728
Roland Burris	363,591

a. Assuming the Blagojevich vote was unchanged, what is the minimum percentage of the Burris votes would Vallas have needed in order to have won a plurality of the votes? (Give your answer to the nearest hundredth of a percent.)

b. Suppose the election had been decided by a runoff between the top two finishers if none of the candidates received a majority of the votes. In a runoff election between Blagojevich and Vallas, what is the minimum percentage of Burris supporters that would have needed to vote for Vallas in order for Vallas to have won the election if all original voters turned out and those who voted for Blagojevich and Vallas voted the same way? (Give your answer to the nearest whole percent.)

2. In a Research Illinois Poll conducted from October 20 through October 23, 2008 a total of 800 likely voters who vote regularly in state elections were interviewed statewide by telephone. Those interviewed were asked, "What do you consider the single most important issue facing Illinois?" The results were as follows:

- A education funding
- B health care
- C balancing the budget
- D rebuilding infrastructure
- E preventing tax increases

	Democrats	Republicans	Independents
rank	339	214	247
1	B	E	A
2	A	C	B
3	C	A	C
4	D	B	D
5	E	D	E

- a. Is there a Condorcet winner for this poll? If so, what is it?
- b. Does the plurality method of picking a winner violate the Condorcet Winner Criterion in this election? Explain.
- c. Which issue wins under Borda count?

3. The Jefferson High School math club is having its end of the year pizza party. In order to decide what kind of pizza to order, the group has decided to have each member prepare a preference list. The results are summarized as follows:

- A veggie
- B pepperoni
- C sausage and mushroom
- D ham and pineapple

Number of Voters					
rank	5	4	2	6	9
1	D	C	C	B	A
2	B	D	B	C	D
3	C	B	D	D	B
4	A	A	A	A	C

a. If the club uses the Hare system to determine the winner, which type of pizza does the group choose?

b. Suppose the group of 2 voters with the preference ranking C changes its ranking to B
B C
D D
A A

If the club now uses the Hare system to determine the winner, which type of pizza will be chosen?

c. What desirable property of voting systems has been violated in this example? Explain.

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Voting Theory SOLUTIONS

1. a. Vallas would have needed 457,198 votes in order to have a plurality, i.e., he would have needed 25,470 additional votes, or 7.01% of the Burriss votes.

b. The total number of votes cast was 1,252,516, of which Blagojevich received only 36.5%. Thus, a runoff would have been required. Let x be the number of Burriss voters supporting Vallas in the runoff. We require

$$431,728 + x > 457,197 + (363,591 - x)$$

$$431,728 + x > 820,788 - x$$

$$2x > 389,060$$

$$x > 194,530$$

Vallas would have needed 194,531 or 54% of the Burriss votes. The final result would have then been

<u>Vallas</u>	<u>Blagojevich</u>
626,259	626,257

2. a. To determine if there is a Condorcet winner, we must perform head-to-head comparisons.

A to B number preferring A: $214 + 247 = 461$
 number preferring B: 339
 outcome: A wins

A to C number preferring A: $339 + 247 = 586$
 number preferring C: 214
 outcome: A wins

A to D number preferring A: $339 + 214 + 247 = 800$
 number preferring D: 0
 outcome: A wins

A to E number preferring A: $339 + 247 = 586$
 number preferring E: 214
 outcome: A wins

Since A wins every head-to-head comparison, A (education funding) is the Condorcet winner.

b. Yes, the plurality method of picking a winner violates the Condorcet Winner Criterion. Under the plurality method, B (health care) was the most important issue facing Illinois. The plurality method failed to pick as winner the Condorcet winner.

c. Using 4 points for a first place ranking, 3 points for a second place ranking, 2 points for a third place ranking, 1 point for a fourth place ranking, and 0 points for a fifth place ranking, we have the following Borda point scores:

issue	computation	total points
A	$339(3) + 214(2) + 247(4)$	2433
B	$339(4) + 214(1) + 247(3)$	2311
C	$339(2) + 214(3) + 247(2)$	1814
D	$339(1) + 214(0) + 247(1)$	586
E	$339(0) + 214(4) + 247(0)$	856

A (education funding) is the most important issue facing Illinois if we make that determination by using the Borda count method.

3. a.

Number of Voters

rank	5	4	2	6	9
1	D	C	C	B	A
2	B	D	B	C	D
3	C	B	D	D	B
4	A	A	A	A	C

The numbers of first place votes are: A=9, B=6, C=6, D=5. Since no choice received a majority of the first place votes, D is eliminated. The preference ranking is now:

Number of Voters

rank	5	4	2	6	9
1	B	C	C	B	A
2	C	B	B	C	B
3	A	A	A	A	C

The numbers of first place votes are: A=9, B=11, C=6. Since no choice has a majority of first place votes, C is eliminated. The preference ranking is now:

Number of Voters

rank	5	4	2	6	9
1	B	B	B	B	A
2	A	A	A	A	B

The numbers of first place votes are: A=9, B=17. B is the winner. The club will order pepperoni pizza.

b. With the change in the group of 2 voters' ranking, the preference ranking becomes:

Number of Voters				
rank	5	4	8	9
1	D	C	B	A
2	B	D	C	D
3	C	B	D	B
4	A	A	A	C

round 1: C is eliminated

Number of Voters				
rank	5	4	8	9
1	D	D	B	A
2	B	B	D	D
3	A	A	A	B

round 2: B is eliminated

Number of Voters				
rank	5	4	8	9
1	D	D	D	A
2	A	A	A	D

The number of first place votes is: A=9, D=17. D is the winner. The club will order ham and pineapple pizza.

c. Monotonicity has been violated. Under the original ranking, pepperoni was the winner. Changing two ballots to place pepperoni in first place rather than second place caused pepperoni to lose.

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Voting Theory – Extemporaneous Questions

1. Explain what is meant by the “Pareto condition”. Name a method of picking a winner which can violate the Pareto condition.

2. True or false? If a candidate receives a majority of the first place votes, that candidate will be a Condorcet winner. Explain.

3. Suppose that in a Borda count election there are n candidates and v voters, with each first place vote worth $n - 1$ Borda points. What is the total number of Borda points to be distributed among the candidates?

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Voting Theory – Extemporaneous SOLUTIONS

1. The Pareto condition requires that if every voter prefers alternative A to alternative B, then B should not be the winner of the election. The method of sequential pairwise comparisons can violate the Pareto condition.

2. True. If candidate A receives a majority of first place votes, then more than 50% of the voters rank candidate A higher than any other candidate. Thus, in each pairwise comparison, candidate A will be preferred by at least 50% of the voters and hence will win that particular pairwise comparison. Winning every pairwise comparison makes candidate A a Condorcet winner.

3. Each voter has $(n-1) + (n-2) + \dots + 3 + 2 + 1 + 0 = \frac{n(n-1)}{2}$ Borda points to allocate among the candidates. Thus, the total number of Borda points to be distributed is $\frac{v \cdot n \cdot (n-1)}{2}$.