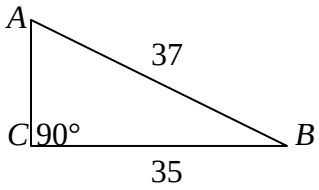
1. Let $E = \{19,\ 29,\ 39,\ 49,\ 59,\ 684,\ 979794,\ 897,\ 923687541,\ 6876532\}$. If one of the members of E is selected at random, find the probability that the number selected is an integral multiple of nine. Express your answer as a common fraction reduced to lowest terms.

2. The first three terms of a geometric sequence are respectively: 1200, 720, 432. Find the fourth term of this geometric sequence. Express your answer as an **exact** decimal.

3. In the diagram with measures as shown, find $\sin(\angle ABC)$. Express your answer as a common fraction reduced to lowest terms.



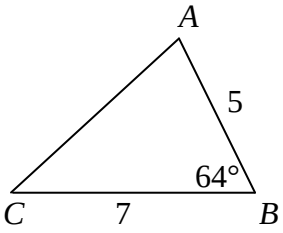
4. **(Multiple Choice)** The conditions for a circle are that it passes through the point $(5,6)$ and is tangent to the two lines whose equations are $3x-4y+1=0$ and $4x+3y=7$. Exactly one of the statements A, B, or C is true. For your response, write the capital letter of the true statement:

   A) No circle is determined.
   B) Exactly one circle is determined.
   C) More than one circle is determined.

   Note: Be certain to write the correct capital letter as your answer.

5. If $(a^2+2b)^{14}$ is expanded and completely simplified, find the numerical coefficient of the term in which the exponent of a is 8.

6. In ΔABC with measures as shown, $(AC)^2 = 7^2 + 5^2 - k\cos(64°)$. Find the value of k.



7. Expressed in interval form, the range of values of x for which the series $\sum_{n=1}^{\infty}\left(\left(\frac{x+1}{3}\right)^n\right)$ is convergent is (k, w). Find the value of $(2k+9w)$.

8. A committee of four is to be chosen from five Whigs and six Tories. How many fewer possibilities are there if there must be two persons from each party on the committee than if there must be at least one person from each party on the committee?

9. Let the brackets [ ] represent the greatest integer function. If $6[\frac{x}{8}]^3 - 19[\frac{x}{8}]^2 - 116[\frac{x}{8}] = -84$, then the set of all possible values for x can be denoted by $\{x : k \le x < w\}$ for some integers k and w. Find the value of $(k+w)$.

10. Urn A contains two orange marbles and one blue marble. Urn B contains two orange marbles and two blue marbles. An urn is selected at random and then one of the marbles is selected at random from that urn. Find the probability that the marble selected is orange. Express your answer as a common fraction reduced to lowest terms.

11. The sum of the reciprocals of two numbers is 20. The sum of the two numbers is 10. Find the product of the two numbers.

12. When $(3x^2 - 2x + 5)(2x - 3)^{15}$ is expanded and completely simplified, find the coefficient of the x^{12} term.

13. Let $(x-1)$, $(x-2)$, and $(x+17)$ be factors of the polynomial $x^3 + kx^2 + wx + p$ where k, w, and p represent integers. Find the value of $(k+p)$.

14. Lee and Cindy have \$200 in their joint checking account. One day each writes a check, both amounts randomly selected from the interval $(0, 200]$. Find the probability that each wrote a check for more than \$60 and that the sum of their two checks is less than \$200. Express your answer as a common fraction reduced to lowest terms.

15. The fifth term of an arithmetic sequence is 12, and the tenth term is 20. Find the third term. Express your answer as a **decimal**.

16. Find the value of k in the determinant equation: $\begin{vmatrix} 5 & 24 \\ 25 & k \end{vmatrix} = \begin{vmatrix} 6 & 40 \\ 13 & 5 \end{vmatrix}$.

17. Let e be the base for natural logarithms and let ln be the symbol for natural logarithm. If $p = \ln 4$, then the value of x such that $e^{(4x)} - e^{(2x-3)} = 12e^{(-6)}$ can be expressed as $\frac{p-k}{w}$. If k and w are positive integers, find the smallest possible value of $(2k + 5w)$.

18. Let $i = \sqrt{-1}$ and let k and w represent real numbers. If $(k + wi)(6 - i) = 15 + 16i$ then find the value of $(k + w)$.

19. The points $(-6,15)$, $(9,15)$, and $(-6,7)$ are the vertices of a triangle. Rounded to the nearest degree, find the degree measure of the smallest angle of this triangle.

20. $A = \{1, 3, 11, 8, x\}$ where the set consists of 5 **different** positive integers. Let $k = \sigma_x$, the total population standard deviation of A. Let w be the median of A. For how many distinct values of x is $|k - w| < 1$?

2009 RA
Pre-Calculus

Name ANSWERS

School _____

(Use full school name – no abbreviations)

_____ **Correct X 2 pts. ea. =** ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{3}{10}$ **(Must be this reduced common fraction.)**

2. 259.2 **(Must be this exact decimal.)**

3. $\frac{12}{37}$ **(Must be this reduced common fraction.)**

4. C **(Must be this capital letter.)**

5. 1,025,024

6. 70

7. 10

8. 160

9. 104

10. $\frac{7}{12}$ **(Must be this reduced common fraction.)**

11. $\frac{1}{2}$ **or** 0.5 **or** .5

12. $-2,946,198,528$

13. 48

14. $\frac{2}{25}$ **(Must be this reduced common fraction.)**

15. 8.8 **(Must be this decimal.)**

16. 22

17. 16

18. 5

19. 28 **(Degrees optional.)**

20. 8