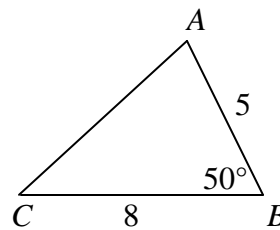


**NO CALCULATORS**

1. Find the value of  $(16^{0.43})(16^{2.07})\left(16^{\frac{1}{4}}\right)$ . Express your answer as an **integer**.
2. Find the maximum **sum** of the distinct members of the largest **proper** subset of  $\{1, 2, 3, 4, 5\}$ .
3. A line passes through the points represented by  $(-3, 7)$  and  $(k, 32)$ . If this line has a direction vector of  $(2, 5)$ , find the value of  $k$ .
4. There are two values of  $x$  for which the three terms:  $3$ ,  $2x+1$ , and  $7x-1$ , taken in that order, will form a geometric sequence. One of those values for  $x$  is an integer. Find the **other** value for  $x$ . Express your answer as a **decimal**.

5. In  $\triangle ABC$  with  
measures as shown,  
 $8^2 + 5^2 - (AC)^2 = k \cos(50^\circ)$ .  
Find the value of  $k$ .



6. An ellipse has the equation of  $\frac{4(x-2)^2}{169} + \frac{(y+3)^2}{16} = 1$ . Find the length of the major axis of this ellipse.
7. Let  $i = \sqrt{-1}$ . Find the exact product of the two distinct roots of the equation  $x^2 = 9i$ .

**NO CALCULATORS**

8. Bob and Judy are playing a game with four dice whose faces are numbered as follows:

Orange Die: 4, 4, 4, 4, 4, 4

Blue Die: 8, 8, 2, 2, 2, 2

Scarlet Die: 7, 7, 7, 1, 1, 1

Gray Die: 6, 6, 6, 6, 0, 0

Bob must select a die first. After Bob selects his die, he rolls that die once. Then Judy must select her die. Judy then rolls her die once. The winner is the one who rolled the higher number. Bob selects the scarlet die and rolls. Judy knows Bob selected the scarlet die, but not the result of his roll. If Judy applies her best strategy, find the probability that Judy will win the game. Express your answer as a common fraction reduced to lowest terms.

9. **(Multiple Choice)** For your answer write the capital letter that corresponds to the best answer.

Which of the following is **not** an arithmetic series?

A)  $\sum_{n=1}^{18} 2n + 1$

B)  $\sum_{a=1}^6 \frac{1}{6} a$

C) The sum of the first 10 integral perfect squares.

D) The sum of the first 10 integral positive multiples of 3.

E) The sum of the first 10 positive integers.

**Note: Be certain to write the correct capital letter as your answer.**

10. Let  $i = \sqrt{-1}$ . If  $|6 + ki| = 6\sqrt{5}$ , find the smallest possible real value of  $k$ .

11. The graph of  $y = \frac{x^3 + 11x^2 + 10x - 72}{x^2 - x - 2}$  has a slant or oblique asymptote of  $y = mx + b$ . Find the value of  $(m + b)$ .

12. One of the angle bisectors of the angles formed by the graphs of  $5x - 12y + 80 = 0$  and  $12x + 5y - 2 = 0$  is represented by the equation  $kx - 7y + w = 0$  where  $k$  and  $w$  are integers. Find the value of  $(k + w)$ .

13. How many distinct numbers are in the **range** of the relation:  $\{(5, 2), (4, 0), (6, 2), (854, 2)\}$ ?

**NO CALCULATORS**

14. If  $0^\circ < x < 45^\circ$ , find the sum of all distinct values of  $x$  such that  $\sin(24x)^\circ + \cos(6x)^\circ = 0$ .
15. The linear regression line (also known as the least squares regression line) for the three points  $(5, 23)$ ,  $(6, 41)$ , and  $(7, 95)$  is  $y = 36x - w$ . Find the value of  $w$ .
16.  $ABCD$  is a convex quadrilateral with  $AB = 4$ ,  $BC = 30$ , and  $CD = 56$ .  $\angle BCD$  is obtuse,  $\cos(\angle ABC) = -\frac{3}{5}$ , and  $\sin(\angle BCD) = \frac{3}{5}$ . Find the length of  $\overline{BD}$ .
17. All ages in this problem are in whole numbers of years. A father has three sons whose ages form an arithmetic progression. The father's age is now equal to the sum of the sons' ages. Five years ago, the sum of the sons' ages at that time was half of the father's present age, and the oldest son then was 4 times as old as the youngest son then. Find the number of years in the present age of the oldest son.
18. Let  $k$  be selected at random from the set:  $\{2, 3, 5, 7, 8\}$ . Find the probability that, for all radian values of  $\theta$ , that  $\cos\left(\frac{5\pi}{k} - \theta\right) = \sin(\theta)$ . Express your answer as a common fraction reduced to lowest terms.
19. If  $x$  and  $y$  are real, and  $x^2 + y^2 = 1$ , find the maximum value of  $(x + y)^8$ .
20. Find the exact value of the diameter of a circle that is circumscribed about a triangle with sides of lengths 11, 31, and 35.

# 2009 AA

School \_\_\_\_\_ **ANSWERS** \_\_\_\_\_

## Jr/Sr 8 Person

(Use full school name – no abbreviations)

\_\_\_\_\_ Correct X **5** pts. ea. =

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. \_\_\_\_\_ 2048 \_\_\_\_\_

11. \_\_\_\_\_ 13 \_\_\_\_\_

2. \_\_\_\_\_ 14 \_\_\_\_\_

12. \_\_\_\_\_ 95 \_\_\_\_\_

3. \_\_\_\_\_ 7 \_\_\_\_\_

13. \_\_\_\_\_ 2 \_\_\_\_\_

4. \_\_\_\_\_ 0.25 or .25 \_\_\_\_\_  
(Must be decimal answer.)

14. \_\_\_\_\_ 113 \_\_\_\_\_  
(Degrees optional.)

5. \_\_\_\_\_ 80 \_\_\_\_\_

15. \_\_\_\_\_ 163 \_\_\_\_\_

6. \_\_\_\_\_ 13 \_\_\_\_\_

16. \_\_\_\_\_ 82 \_\_\_\_\_

7. \_\_\_\_\_  $-9i$  or  $0 - 9i$  \_\_\_\_\_

17. \_\_\_\_\_ 13 \_\_\_\_\_  
(Years optional.)

8. \_\_\_\_\_  $\frac{2}{3}$  \_\_\_\_\_  
(Must be this reduced common fraction.)

18. \_\_\_\_\_  $\frac{1}{5}$  \_\_\_\_\_  
(Must be this reduced common fraction.)

9. \_\_\_\_\_ C \_\_\_\_\_  
(Must be this capital letter.)

19. \_\_\_\_\_ 16 \_\_\_\_\_

10. \_\_\_\_\_ -12 \_\_\_\_\_

20. \_\_\_\_\_  $\frac{62\sqrt{3}}{3}$  or  $20.\bar{6}\sqrt{3}$  \_\_\_\_\_  
(Must be this exact answer.)