Oral Competition **ICTM Regional, Division AA 2009**

GRAPHS AND NETWORKS

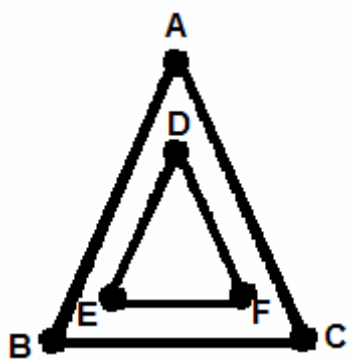
Ch. 1, For All Practical Purposes, sixth edition

1) Explain the following, in your own words:

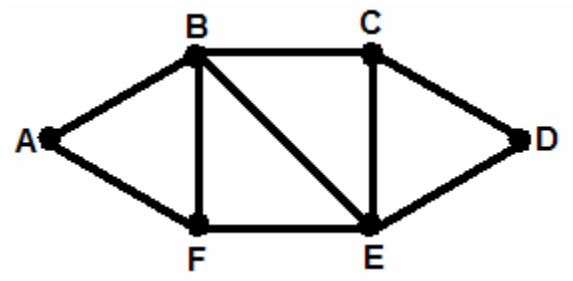
a) What is an Euler circuit?
b) What does it mean to Eulerize a graph? Give an example.
c) What is the valence number for a vertex? Give an example.
d) Describe the valence numbers in a graph which has an Euler circuit.

2) Which graphs have Euler circuits? For the one(s) that do, find the Euler circuits by numbering the edges in the order the Euler circuit uses them starting at point A. For the one(s) that doesn't (don't) explain why no Euler circuit is possible.

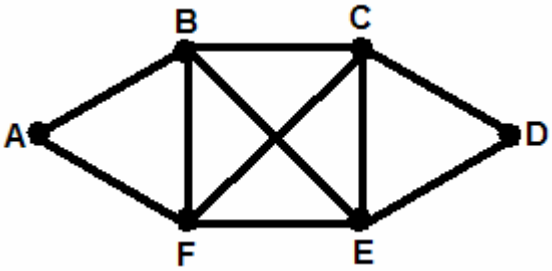
a)



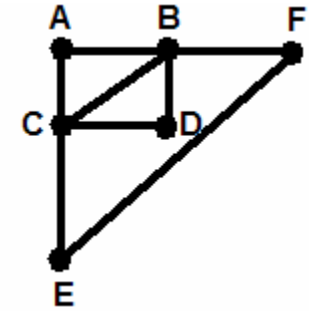
b)



c)



d)



3) If a rectangular street is *r* blocks by *s* blocks, find a formula for the minimum number of edges that must be added to Eulerize a graph representing the network in terms of *r* and *s* when both *r* and *s* are > 1. Determine a formula when

a) both *r* and *s* are odd . Test your formula with the case: 5 blocks by 5 blocks using the edge walker technique.
b) both *r* and *s* are even. Test your formula with the case: 6 blocks by 6 blocks using the edge walker technique.
c) *r* is odd and *s* is even. Test your formula with the case:
5 blocks by 4 blocks using the edge walker technique.

Solutions for the Questions for the Oral Competition

ICTM Regional, Division AA 2009

GRAPHS AND NETWORKS

Ch. 1, For All Practical Purposes, sixth edition

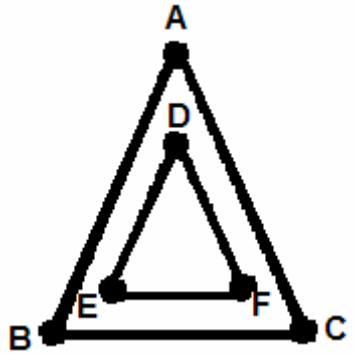
1) a) An Euler circuit is a circuit that traverses each edge of a graph exactly once.

b) Eulerizing a graph is adding new edges to a graph so as to make it a graph that possesses an Euler circuit. The new edges added must be duplicates of existing edges.

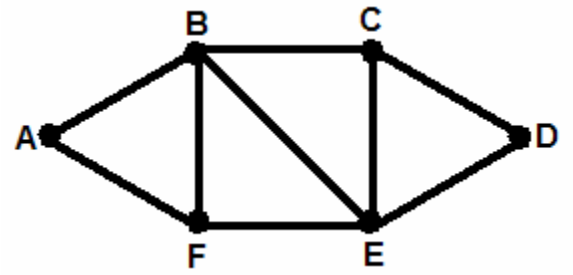
c) The valence of a vertex is the number of edges that touch the vertex.

d) All of the valences need to be even numbers greater than 1.

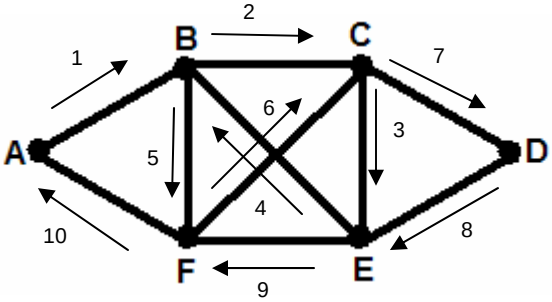
2) a) Does not have an Euler circuit.
It is not a connected graph.



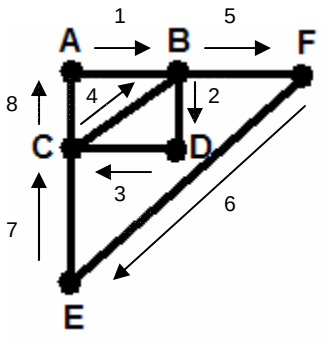
b) Does not have an Euler circuit.
Vertices C and F do not have an even valence.



c) Graph has an Euler circuit.
There are several possibilities.
One is ABCEBFCDEFA



d) Graph has an Euler circuit.
There are several possibilities.
One is ABDCBFECA.



3) If a rectangular street is *r* blocks by *s* blocks, find a formula for the minimum number of edges that must be added to Eulerize a graph representing the network in terms of *r* and *s* when both *r* and *s* are >1. Determine a formula when

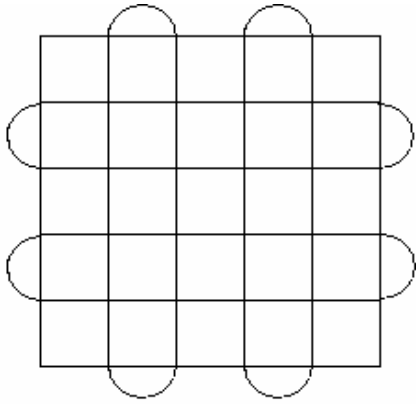
a) both *r* and *s* are odd . Test your formula with the case: 5 blocks by 5 blocks using the edge walker technique.

b) both *r* and *s* are even. Test your formula with the case: 6 blocks by 6 blocks using the edge walker technique..

c) either *r* or *s* is even and the other one is odd. Test your formula with the case: 5 blocks by 4 blocks using the edge walker technique..

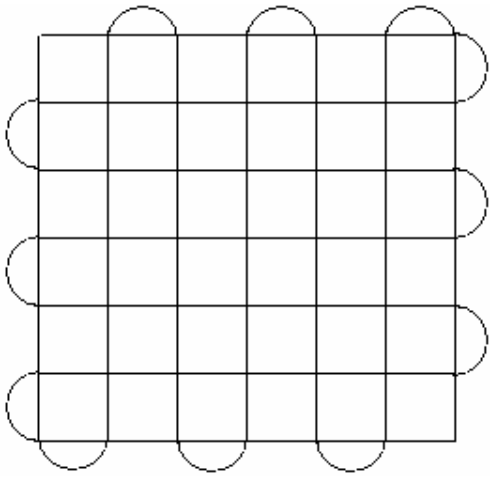
a) If *r* and *s* are odd, where $r = 2a+1$ and $s = 2b+1$ (*a* and *b* positive integers which are at least 1) then a formula for the number of repeated edges is $2(a + b)$.

For 5 blocks by 5 blocks, $r = 2a+1$ so $a = 2$ and $s = 2b+1$ so $b = 2$; therefore, the minimum number of repeated edges is $2(a+b) = 2(2 + 2)= 8$.



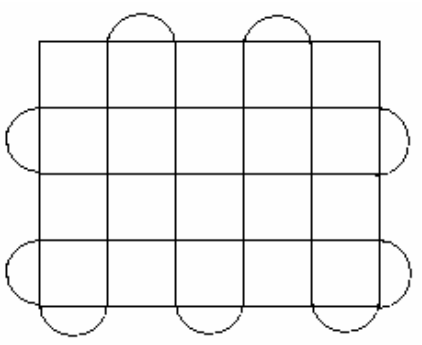
b) If *r* and *s* are even, where $r = 2a$ and $s = 2b$ (*a* and *b* positive integers which are at least 1) then a formula for the number of repeated edges is $2(a + b)$.

For 6 blocks by 6 blocks, $r = 2a$ so $a = 3$ and $s = 2b$ so $b = 3$; therefore, the minimum number of repeated edges is $2(a + b) = 2(3 + 3) = 12$.



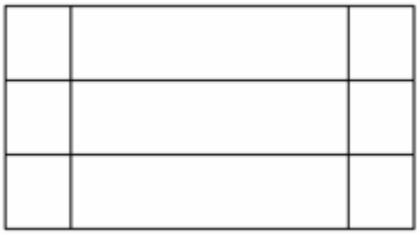
c) If r is odd and s is even, where $r = 2a+1$ and $s = 2b$ (a and b positive integers which are at least 1) then a formula for the number of repeated edges is $2(a + b) + 1$.

For 5 blocks by 4 blocks, $r = 2a + 1$ so $a = 2$ and $s = 2b$ so $b = 2$; therefore, the minimum number of repeated edges is $2(a + b) + 1 = 2(2 + 2) + 1 = 9$.



Extemporaneous Questions – Division AA Oral - ICTM Regionals 2009

1) In the figure below, all blocks are 1000 by 1000 feet, except for the middle column of blocks, which are 1000 by 4000 feet.

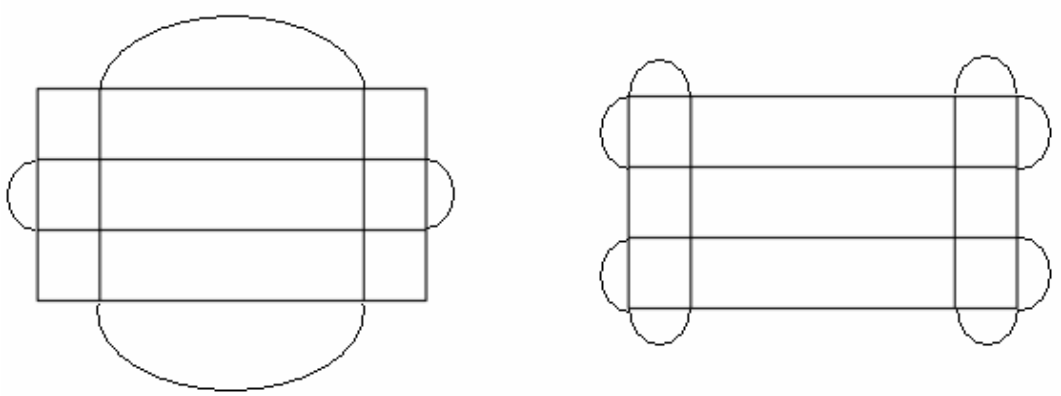


a) Draw an Euler circuit on this graph, or Eulerize if needed.
b) Determine the minimum distance that needs to be added to the graph so that it has an Euler circuit.
c) What is the minimum total distance travelled if someone were to travel an Euler circuit on this graph?

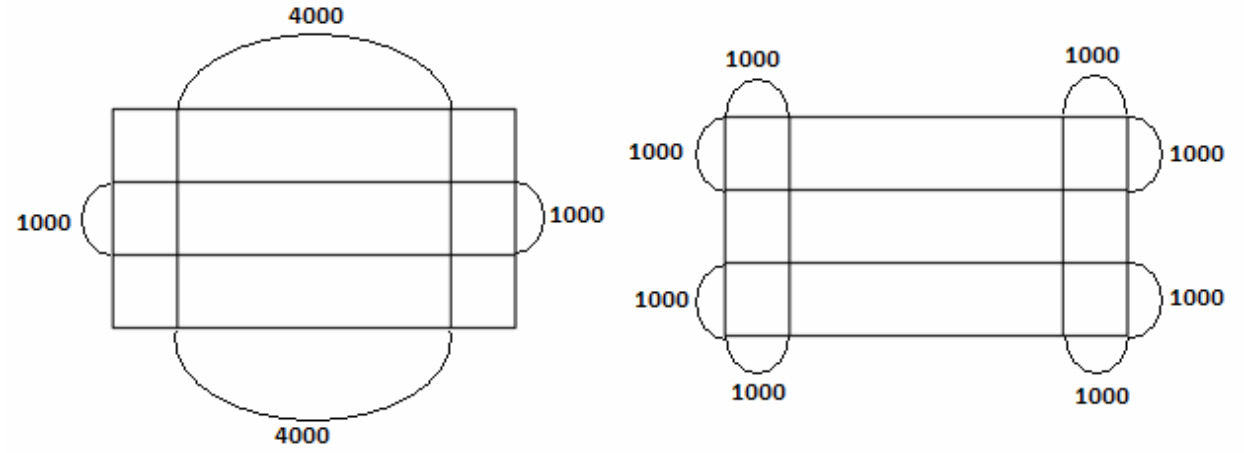
2) The text mentions practical uses for developing Euler circuits of checking parking meters, snow plowing, collecting garbage, and delivering mail. What other real world applications of Euler circuits come to your mind?

Solutions for Extemporaneous Questions
Division AA Oral - ICTM Regional 2009

1) a) There are two basic ways to Eulerize this circuit.



b) T he graph on the left needs to have 10,000 ft added to it in order to Eulerize the graph; whereas, the graph on the right needs to have 8,000 ft added it to it in order to Eulerize the graph..



d) If one were to travel the circuit, the least total distance is 44,000 ft..
The distance of the original graph is 36,000 ft.

Total distance = original distance + distances for added edges
For the circuit on the left, the total distance is 36,000 + 10,000 = 46,000 ft.
For the circuit on the right, the total distance is 36,000 + 8,000 = 44,000 ft.

2) There will be a variety of responses. Consider those that are non-traditional to be worth more than the obvious ones.