1. $\begin{bmatrix} 3 & 2 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} k \\ w \end{bmatrix}$. Find the value of $(2k+3w)$.

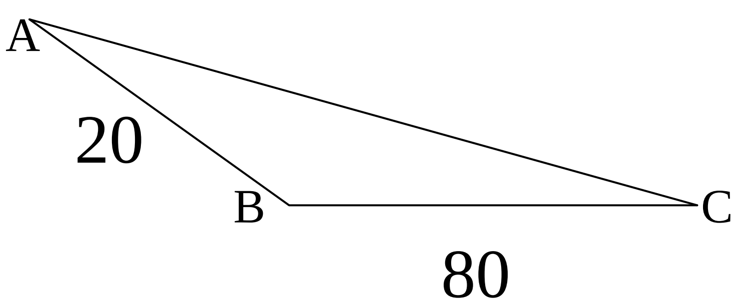
2. Given the set: $\{\log_2(16),\ \log_3(16),\ \log_4(16),\ \log_5(16)\}$. If one of the 4 members of the set is drawn at random, find the probability that the member drawn could represent a positive integer. Express your answer as a common fraction reduced to lowest terms.

3. Find the ordered pair that represents the sum of the following two vectors: $(-5,6)$ and $(17,7)$.

4. Let n be a positive integer. For $n \geq 1$, $x_{(n+2)} = x_{(n)} + x_{(n+1)}$. If $x_1 = 3$ and $x_2 = 1$, find the value of x_{10}.

5. Matrix $[A]$ contains 23 rows and 13 columns. Matrix $[B]$ contains 13 rows and 92 columns. Find the number of rows in the product matrix $[A][B]$.

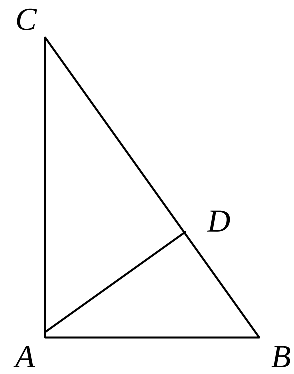
6. In ΔABC as with side-lengths as shown, $\angle ABC = 120°$. Find the exact length of $\overline{AC}$.



7. A woman is sitting in the stands behind one end zone of a football field. She observes, in the same vertical plane with her position, two players standing on their own goal lines, exactly 100 yards apart. Looking at the spots on the ground where they are standing, a player at one goal line is at an angle of depression of 15° and a player at the other goal line is at an angle of depression of 8°. Assuming the football field is horizontal, find the **number of feet** in the vertical height of the woman's eye above the horizontal plane of the football field. Express your answer as a **decimal** rounded to the nearest hundredth of a foot.

8. If three real geometric means are inserted between 15 and $\frac{5}{27}$, find the value of the middle term. Express your answer as an improper fraction reduced to lowest terms.

9. In the diagram ΔCAB is a right triangle with $\angle CAB = 90°$. Point D is on $\overline{BC}$ such that $\overline{AD} \perp \overline{BC}$. Let the lengths of ΔCAB be a, b, and c with these lengths respectively opposite the angles with vertices at A, B, and C. If $b \sin C\left(b \cos C + c \cos B\right) = 41$, and if the length of $\overline{AD}$ is an integer, find the largest possible value of AD.



10. The roots of $x^3 + kx^2 - 59x + 420 = 0$ are 5, -7, and 12. Find the value of k.

11. When $(2x-3)^8$ is expanded and completely simplified, find the numerical coefficient of x^5.

12. The first term of an infinite geometric sequence of real terms is 225, and the fourth term of this geometric sequence is 14.4. Find the sum of the terms of this infinite geometric sequence.

13. The sum of the terms of an arithmetic sequence with 13 terms and common difference of 3 is found to be 286. Find the thirteenth term of this sequence.

14. Let there be k objects, each of weight w. When these objects are weighed in pairs, the sum of the weights of all possible pairs is 770. When these objects are weighed in groups of four, the sum of the weights of all possible groups of four is 9240. Find the value of $\left(k + w\right)$.

15. Let $i=\sqrt{-1}$. Find the exact value of $\left|\sqrt{6}+i\sqrt{8}\right|$. Write your answer in simplified radical form.

16. Tom is a gambler. He has a pair of fair, standard cubical dice. He selects two different numbers in advance from the face numbers 1, 2, 3, 4, 5, and 6. He then rolls the pair of dice. If both numbers showing match his two selections, he wins \$108. If exactly one number showing matches one of his two selections, he neither wins nor loses. If neither number showing matches either of his two selections, he loses \$54. Find Tom's mathematical expectation each time he rolls the pair of dice. Express your answer by including the word "lose" or the word "win." For example, your answer might be "lose \$2" or might be "win \$7."

17. Let D be the point that is $\frac{1}{3}$ of the way from B to A for equilateral triangle ΔABC. Let P be a point on $\overline{BC}$ such that $AP+PD$ is the smallest possible value. If the perimeter of ΔABC is 63, find the exact value of $\left(AP+PD\right)$.

18. Let $i=\sqrt{-1}$ and let k and w represent real numbers. If $(k+wi)(4+i)=33-13i$ then find the value of $\left(k+w\right)$.

19. Find the acute angle between the normal vectors of the following two planes:

$$3x+2y+5z+4=0 \text{ and } -2x-3y+6z-2=0$$

Round your answer to the nearest minute and then express your answer in degrees and minutes in the form: $k°w'$.

20. Points A, B, C, and D are the vertices of a rectangle. Point E is in the interior of this rectangle. $AE=8$, $EC=21$. The lengths of $\overline{DE}$, $\overline{BE}$, and $\overline{BC}$ are integers. If $DE<BE$, find the largest possible value for the length of $\overline{BC}$.

2009 RAA
Pre-Calculus

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 68
2. $\frac{1}{2}$ (Must be this reduced common fraction.)
3. $(12,13)$ (Must be this ordered pair.)
4. 97
5. 23
6. $20\sqrt{21}$ (Must be this exact simplified radical.)
7. 88.67 (Must be this decimal, feet optional.)
8. $\frac{5}{3}$ (Must be this reduced improper fraction.)
9. 4
10. -10
11. -48384
12. 375
13. 40
14. 18
15. $\sqrt{14}$ (Must be this exact simplified radical.)
16. Lose 18 (Must have "lose", not -18, $ or dollars optional.)
17. $7\sqrt{13}$ (Must be this exact simplified radical.)
18. 2
19. $65°21'$ (Must be in the form Degrees and minutes.)
20. 19