

ICTM 2009 DIVISION A STATE FINALS

1. If $5^{(5x-3)} = 25^{(2x+3)}$, find the value of x .

2. Find the value of the expression $-2(7-3x) - [8x - (3-x)]$ when $x = \text{ANS}$.

3. Solve for k if $\begin{vmatrix} 1 & 0 & -10 \\ k & 0 & 0 \\ 0 & 12 & 0 \end{vmatrix} = \text{ANS}$. Write your answer as a common fraction reduced to lowest terms.

4. ANS should be in the form $\frac{k}{w}$. $f(x) = ax^2 + bx + c$ is a quadratic function with a zero at $x = 2$, and the graph of $y = f(x)$ contains the points $(4, k)$ and $(5, w)$. Find the sum $(a + b + c)$.

ANSWERS:

1. 9

2. -38

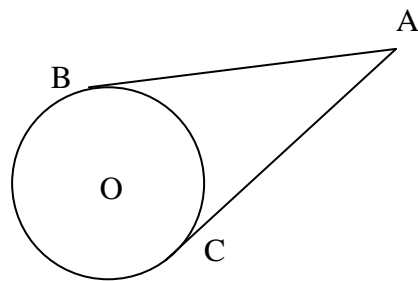
3. $\frac{19}{60}$ (Must be this reduced proper fraction.)

4. 22

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- Set $C = \{x\}$. Set C has 2 subsets: $\emptyset, \{x\}$.
 Set $D = \{2,3\}$. Set D has 4 subsets: $\emptyset, \{2\}, \{3\}, \{2,3\}$.
 Set $E = \{a,b,c\}$. Set E has 8 subsets: $\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$.
 Set A has 96 more subsets than Set B . How many **more** elements are in Set A than in Set B ?

- Given Circle O with \overline{AB} and \overline{AC} tangent segments to the circle. $AB = 2x - 3y$ and $AC = 8 - xy$. Find the value of x when $y = ANS$. Write your answer as an improper fraction reduced to lowest terms.



- Let $k = ANS$. $f(x)$ is a function such that $f(kx) = 2x + 1$ and $f(t) = 25$. Find the value of t .
- Let e be the base for the natural logarithms (\ln). The exponential equation $e^{2x} = ANS - e^x$ has an exact solution of the form $x = \ln k$. Find the value of k .

ANSWERS:

- 2
- $\frac{7}{2}$ (Must be this reduced improper fraction.)
- 42
- 6

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1. If x represents a real number, find the least possible value for the sum $|x-3|+|x|+|x+3|$.
2. Let $k = 12$ (ANS). Find the sum of the roots for x of the fifth degree equation $(x-2400)(x-k)(x^3-28x^2+77x-50)=0$
3. An investor has \$10,000 to invest. He invests half in a guaranteed deposit that will earn 5% APR. The other half he invests in a speculative but potentially high-yield land deal. The investor's goal is to receive ANS dollars at the end of the first year in total interest from the two halves of his \$10,000 invested. What rate of return must the investor receive on the land deal in order to achieve this goal? The answer is $k\%$ APR where k is a whole number. Write the value of k on your answer sheet. **Do not include “%” in your answer.**
4. $\log_2 3 = M$, $\log_2 5 = N$, $\log_2 7 = P$ and R is a real number constant. Let $k = ANS$. $\log_2 \left(\frac{70k}{8} \right)$ can be written in the form $aM + bN + cP + R$. Find the sum $(a + b + c + R)$.

ANSWERS:

1. 6
2. 2500
3. 45 (Must not have “%” sign.)
4. 3

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1. $P(x)$ is a polynomial in x . If $(3x^4 - 7x^3 + 5x^2 - 4x + 2) - P(x) = x^4 + 6x^3 + x^2 - 5$, what is the coefficient of the degree 3 term of $P(x)$?
2. Let r and t represent the solutions to $x^2 - 5x + ANS = 0$. Find the value of $\frac{2}{rt^2} + \frac{2}{r^2t}$. Write your answer as a common fraction reduced to lowest terms.
3. Find the value of the eccentricity of the conic $(ANS)x^2 - \frac{10}{27}y^2 = 1$. Write your answer as a common fraction reduced to lowest terms.
4. ANS should be an improper fraction reduced to lowest terms in the form $\frac{k}{w}$. Let $p = k + w - 1$. Let $f(x) = 3 + 7 \tan(13x + p)$. Find the negative value for x closest to zero for which $f(x) = 3$. Write your answer as a decimal correct to **4 decimal places**.

ANSWERS:

1. -13
2. $\frac{10}{169}$ (Must be this reduced common fraction.)
3. $\frac{14}{13}$ (Must be this reduced improper fraction.)
4. -0.0667 or $-.0667$ (Must be this exact decimal.)

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1. How many positive integers k leave a remainder of 5 when 65 is divided by that integer k ?
2. Let $k = \text{ANS} + 24$. Find the sum of the first k terms of the increasing sequence 1, 6, 11, 16, 21, 26, \dots where each term is one more than a multiple of 5.
3. a and b are integers in polynomial $P(x)$. Applying the Rational Root Theorem, $P(x) = 19x^7 + 8x^6 + ax^5 - bx^3 + 3x^2 + 2x + \text{ANS}$ has potential rational roots (which may or may not be actual roots.) Find the sum of the positive potential rational roots that are integers.
4. Let $p = \frac{\text{ANS}}{28}$. When written in completely simplified standard form and in decreasing powers of x , the fourth term of $\left(\frac{4}{3}x + ky\right)^{10}$ has numeric coefficient p . Find the exact value of k . Write your answer as a common fraction reduced to lowest terms.

ANSWERS:

1. 7
2. 2356
3. 4480
4. $\frac{9}{16}$ (Must be this simplified common fraction.)