1. Let $C(n,k)=\dfrac{n!}{k!(n-k)!}$. Find the value of $C(12,4)$.

2. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

   If h, k, and j are real numbers such that $h>k$ and $k>j$, then $k+j>h+k$.

3. Given are the following four algebraic fractions: $\dfrac{3}{x-2}$, $\dfrac{16-4x}{2}$, $\dfrac{16}{x^2-5x+6}$, and $\dfrac{15}{x}$. From the set $\{1,2,3,4\}$, one number is selected at random and substituted for x in every one of the four fractions. Find the probability that the value of at least one of the four fractions will be undefined. Express your answer as a common fraction reduced to lowest terms.

4. The equation of a hyperbola whose foci are $(8,0)$ and $(-8,0)$ and whose vertices are $(3,0)$ and $(-3,0)$ can be expressed in the form $\dfrac{x^2}{k}-\dfrac{y^2}{w}=1$. Find the value of $(2k+3w)$.

5. The arithmetic mean of 23, 30, 16, 33, and two other numbers that differ by 2 is 21. If the smallest of these six numbers is discarded, find the arithmetic mean of the remaining five numbers.

6. If $f(x)=5(2x-3)^4+(x+2)^3-(x-5)^2+17$, find the value of $f(5)$.

7. The fifth term of an arithmetic sequence is -18, and the sum of the first thirty-two terms is 1448. Find the ninth term.

8. Find the **sum** of all negative integers that are members of the solution set for x given $|1-2x|<15$.

9. Each person in a room writes down an integer at random from one of the 100 integers from 1 to 100 inclusive. Find the minimum number of persons in the room such that the probability that at least two persons wrote down the same number exceeds 90%.

10. If $f(x)=2^{(2x+1)}$ and $\left(f\left(2-x\right)\right)\left(f\left(x\right)\right)\left(f\left(2+x\right)\right)=kf\left(x\right)$, find the value of k.

11. Let 2 be a root for x of the quartic equation: $ax^4+bx^3+cx^2+bx+a=0$. All the roots are real, the sum of the roots is $\frac{26}{3}$, and a, b, and c are integers. Find the smallest possible value of $\left(a+b+c\right)$.

12. Find the shortest distance from the origin to the line whose equation is $3x-4y=20$.

13. Find the value of $\prod_{x=5}^{15624}\log_x(x+1)$.

14. By substituting 1, 2, 3, 4, and 5 for x in order in a polynomial function in x, the first five terms are respectively: -1, 6, 17, 32, and 51. If $P(x)$ is the polynomial function of lowest degree satisfying the given, find $P(243)$.

15. Given the following seven points: $(8,2)$, $(10,-3)$, $(7,5)$, $(13,-4)$, $(-8,-5)$, $(-3,-2)$, $(-1,1)$. If three of these seven points are selected at random without replacement, find the probability that at least two of the points selected lie below the *x*-axis. Express your answer as a common fraction reduced to lowest terms.

16. If $f(x)=7x+9$ and $g(x)=x^2-3$, find $g(f(-11))$.

17. If $x^2+y^2=7$ and $x^3+y^3=10$, find the largest possible value of $(x+y)$.

18. Given the following system: $\begin{cases} x+y+z+w=14 \\ x^2+y^2+z^2+w^2=54 \\ x^3+y^3+z^3+w^3=224 \\ xyzw=120 \end{cases}$. If x, y, z, and w are positive integers and (x,y,z,w) is a member of the solution set for the given system, find the second least possible value of $(6x+5y+4z+3w)$.

19. Let $i=\sqrt{-1}$. If $(k+3i)(-7+2i)=11i-118$, find the value of k.

20. Let k be a positive integer such that $48<k<90$. Find the sum of all possible distinct values of k such that the product of all distinct positive integral divisors of k is k^2.

2009 SAA
Algebra II

Name ANSWERS

School _____
(Use full school name – no abbreviations)

_____ **Correct X 2 pts. ea. =** ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 495

2. Never **(Must be the whole word.)**

3. $\frac{1}{2}$ **(Must be this reduced common fraction.)**

4. 183

5. 23

6. 12,365

7. 4

8. –21

9. 22

10. 1024

11. 117

12. 4

13. 6

14. 118,337

15. $\frac{22}{35}$ **(Must be this reduced common fraction.)**

16. 4621

17. 4

18. 59

19. 16

20. 908