

ICTM 2009 DIVISION AA STATE FINALS

1. $13! = (2^k)(3^w)(5^2)(7)(11)(13)$. Find the value of $(k + w)$.
2. A bag contains only red and blue marbles. This bag contains 18 red marbles and *ANS* blue marbles. If 2 marbles are drawn at random without replacement from this bag, find the probability that exactly 1 blue marble was drawn. Express your answer as a common fraction reduced to lowest terms.
3. *ANS* should be a common fraction reduced to lowest terms in the form $\frac{a}{b}$. Let $m = a - 24$. The exact distance between the vertices of the parabolas represented by $y = x^2 + 1$ and $y = x^2 + mx + 1$ can be represented in simplest form as $\frac{k\sqrt{w}}{p}$. Report the sum $(k + w + p)$ as your answer.
4. The EPA tests vehicle emissions by rotating tires on a drum system to simulate driving speed. If tires 26 inches in diameter are rotated *ANS* times per minute, what speed, in miles per hour, is the tread of the tire being tested? Round your answer to the nearest mile per hour.

ANSWERS:

1. 15
2. $\frac{45}{88}$ (Must be this reduced common fraction.)
3. 470
4. 36 (Miles per hour optional.)

ICTM 2009 DIVISION AA STATE FINALS

1. Find the value of k if $(3k + 2, 9 - k)$ is a point on the line represented by $\frac{1}{2}x - \frac{1}{3}y - 20 = 0$.
2. Let $k = \text{ANS}$. Find the value of $\frac{(2k+1)!}{(2k-1)!}$.
3. Find the remainder when $P(x) = 2x^3 - 12x^2 + 24x - \text{ANS}$ is divided by $x + 2$.
4. A charity sells exactly 1000 tickets at \$1000 per ticket in a raffle. First prize is a diamond necklace worth \$154,000. Second prize is a \$49,000 car. There are 8 cash third prizes, and each of these third prizes is for the same amount and of lesser value than either the first or second prize. There are no other prizes, and no ticket may win more than one prize. If the expected value of each ticket is ANS dollars, find the dollar value of each of the 8 cash prizes awarded.

ANSWERS:

1. 12
2. 600
3. -712
4. 10625 (\$ or dollars optional.)

ICTM 2009 DIVISION AA STATE FINALS

1. Using only the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 and allowing for repetition of digits, how many 3-digit numbers can be formed in which the digit 7 appears at least once?
2. Let k be an integer and let k^2 have the largest possible value less than ANS . A rhombus has one diagonal that is twice the length of the second diagonal. The area of the rhombus is k^2 . Find the exact perimeter of the rhombus.
3. ANS should be in the simplified radical form $p\sqrt{w}$. Let $f(x) = 28x^2 - 7$ and $g(x) = 5x + k$. Find the positive value of k so that the graph of $(f \circ g)(x)$ intersects the y -axis at $(0, 2p)$. Express your answer as a **decimal**.
4. Let $k = ANS$. Find the exact distance from the point $P(2, 4, 3)$ to the plane $2x + ky + 6z + 11 = 0$.

ANSWERS:

1. 217
2. $28\sqrt{5}$ (Must be this exact answer.)
3. 1.5 (Must be this decimal.)
4. 6

ICTM 2009 DIVISION AA STATE FINALS

1. How many positive integers k leave a remainder of 5 when 65 is divided by that integer k ?
2. Let $k = \text{ANS} + 24$. Find the sum of the first k terms of the increasing sequence $1, 6, 11, 16, 21, 26, \dots$ where each term is one more than a multiple of 5.
3. a and b are integers in polynomial $P(x)$. Applying the Rational Root Theorem, $P(x) = 19x^7 + 8x^6 + ax^5 - bx^3 + 3x^2 + 2x + \text{ANS}$ has potential rational roots (which may or may not be actual roots.) Find the sum of the positive potential rational roots that are integers.
4. Let $p = \frac{\text{ANS}}{28}$. When written in completely simplified standard form and in decreasing powers of x , the fourth term of $\left(\frac{4}{3}x + ky\right)^{10}$ has numeric coefficient p . Find the exact value of k . Write your answer as a common fraction reduced to lowest terms.

ANSWERS:

1. 7
2. 2356
3. 4480
4. $\frac{9}{16}$ (Must be this simplified common fraction.)

ICTM 2009 DIVISION AA STATE FINALS

1. $P(x)$ is a polynomial in x . If $(3x^4 - 7x^3 + 5x^2 - 4x + 2) - P(x) = x^4 + 6x^3 + x^2 - 5$, what is the coefficient of the degree 3 term of $P(x)$?
2. Let r and t represent the solutions to $x^2 - 5x + ANS = 0$. Find the value of $\frac{2}{rt^2} + \frac{2}{r^2t}$. Write your answer as a common fraction reduced to lowest terms.
3. Find the value of the eccentricity of the conic $(ANS)x^2 - \frac{10}{27}y^2 = 1$. Write your answer as an improper fraction reduced to lowest terms.
4. ANS should be an improper fraction reduced to lowest terms in the form $\frac{k}{w}$. Let $p = k + w - 1$. Let $f(x) = 3 + 7 \tan(13x + p)$. Find the negative value of x closest to zero for which $f(x) = 3$. Write your answer as a decimal correct to **4 decimal places**.

ANSWERS:

1. -13
2. $\frac{10}{169}$ (Must be this reduced common fraction.)
3. $\frac{14}{13}$ (Must be this reduced improper fraction.)
4. -0.0667 or $-.0667$ (Must be this exact decimal.)