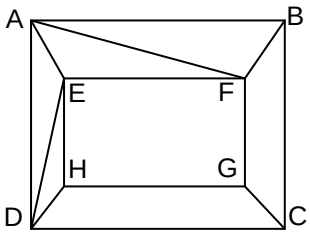
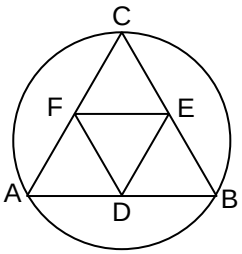
**Questions for the Oral Competition – State AA Level, 2009**
**Ch. 1, 2, For All Practical Purposes, sixth edition**

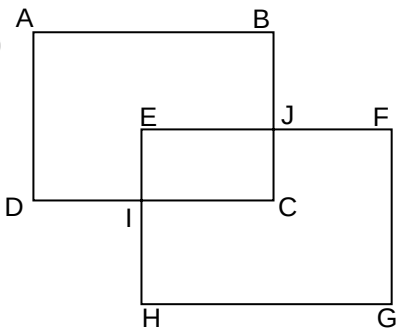
**Answers for the Questions for the Oral Competition – State AA Level, 2009**

1. In your own words, explain the difference between an Euler circuit and a Hamiltonian circuit.

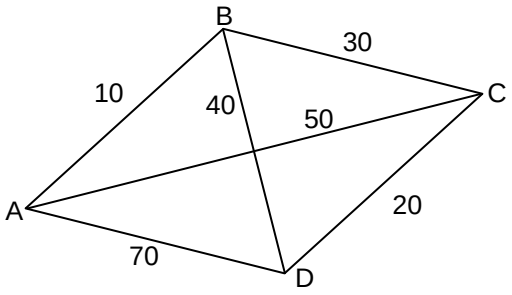
   Determine whether each of the figures below contains an Euler circuit and/or a Hamiltonian circuit.

a)



b)


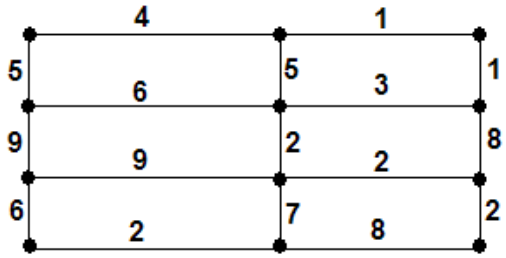

c)



2. a. Use the algorithms listed to solve the traveling salesman problem shown in the graph below starting with vertex A. Explain the procedures that you used for each method.
   i. brute force
   ii. nearest neighbor
   iii. sorted edges

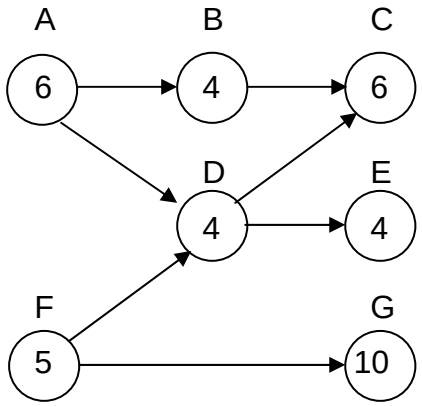



   b. For each of the methods that you used, would the length of the path be different if you started at vertex B? Explain your reasoning.

3. Use Kruskal's algorithm for determining a minimum –cost spanning tree on the graph below. What is the cost of the tree found? Explain how you determined it.




4. a. Give the minimum completion time and critical paths for the order-requirement diagraph below.




b. Reducing the time on one task would create four critical paths. Reducing the time on which task would accomplish this? What would be the new minimum completion time?

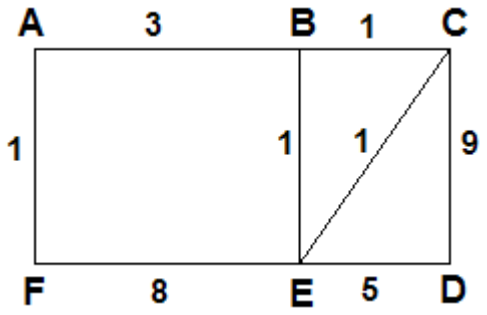
Extra Questions

1. What is the difference between a spanning tree and a Hamiltonian circuit?

2. State one advantage and one disadvantage to a heuristic algorithm.

3. There are six juniors and six seniors on your math team including you. Each person lives at a different location. You want to plan to visit each of your teammates and return home. If it takes ½ minute to compute the total length of one tour, set up (but do not evaluate) the expression you would use to determine how long it will it take to apply the brute force algorithm to find the optimal tour.

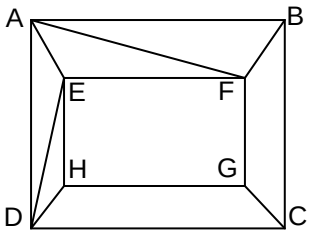
4. It is possible to have more than one spanning tree with the same minimum cost in one graph. Find two different minimum-cost spanning trees for the graph below.

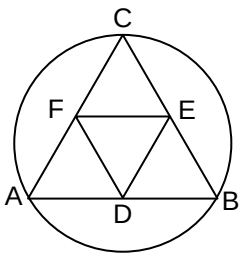


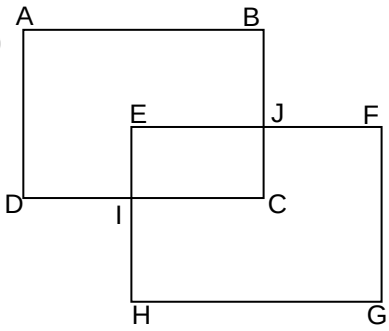
**Answers for the Questions for the Oral Competition – State AA Level, 2009**

1. In your own words, explain the difference between an Euler circuit and a Hamiltonian circuit.
   An Euler circuit must cover each edge only once and return to its starting location. A Hamiltonian circuit must visit each vertex once and return to its starting location.

   Determine whether each of the figures below contains an Euler circuit and/or a Hamiltonian circuit.

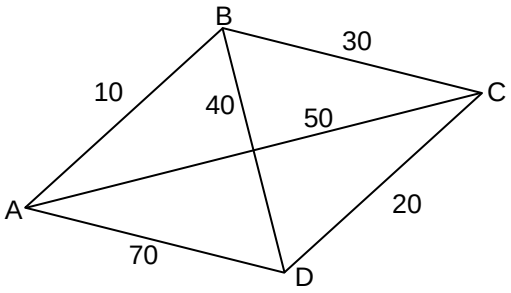
a) 


b) 


c) 


   a) No Euler, Yes Hamiltonian
   b) Yes Euler, Yes Hamiltonian
   c) Yes Euler, No Hamiltonian

2. a. Use the algorithms listed to solve the traveling salesman problem shown in the graph below starting with vertex A. Explain the procedures that you used for each method.
   i. brute force
   ii. nearest neighbor
   iii. sorted edges




i) ABCDA = 10 + 30 + 20 + 70 = 130
   ABDCA = 10 + 40 + 20 + 50 = 120 optimal route
   ACBDA = 50 + 30 + 40 + 70 = 190

ii) At each vertex choose the path that has the least cost. This gives us path ABCDA = 130.

iii) Sort the edges from least cost to most cost. This gives us AB = 10, CD = 20, BC = 30, BD = 40, AC = 50, AD = 70. Use AB, CD, BC. That leaves us with having to use AD to complete the circuit, so the path is ABCDA = 130

b. For each of the methods that you used, would the length of the path be different if you started at vertex B? Explain your reasoning.

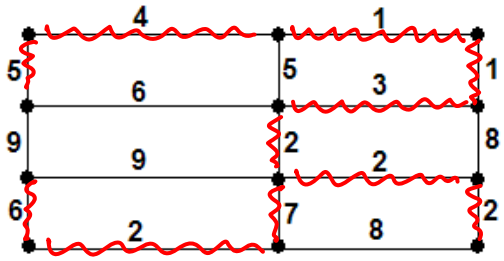
Brute force – No. We already have an optimal route.
BACDB = 120 optimal BCDAB = 130 BDACB = 190
(or explain why the length would not change)

Nearest Neighbor – Yes, if we started at vertex B, we would have an optimal route of 120.

Sorted edges – No. We have already used the three shortest edges.

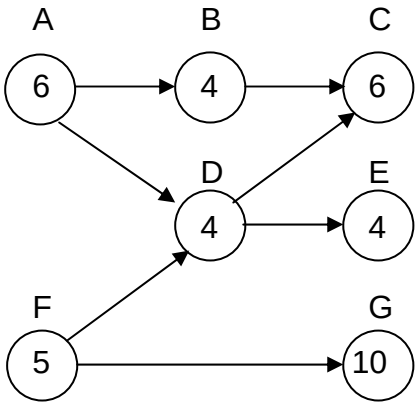
3. Use Kruskal's algorithm for determining a minimum –cost spanning tree on the graph below. What is the cost of the tree found? Explain how you determined it.

Use the least costs first. Continue using the smallest costs unless a circuit is formed. The costs are 1, 1, 2, 2, 2, 2, 3, 4, 5, 5, 6, 6, 7, 8, 8, 9, 9. The squiggly lines show the minimum-cost spanning tree with a cost of 35.




**4 + 1 + 5 + 3 + 1 + 2 + 2 + 6 + 2 + 7 + 2 = 35**

4. a. Give the minimum completion time and critical paths for the order-requirement diagraph below.




Possible paths are:
ABC – length 16
ADC – length 16
ADE – length 14
FDC – length 15
FDE – length 13
FG – length 15

Minimum completion time is 16 with critical paths ABC and ADC.

b. Reducing the time on one task would create four critical paths. Reducing the time on which task would accomplish this? What would be the new minimum completion time?

Reducing Task A by one (to a value of 5) would create a length of 15 for ABC and ADC. Thus, the new minimum completion time would be 15 with four critical paths (ABC, ADC, FDC, and FG)

Answers for the Extra Questions

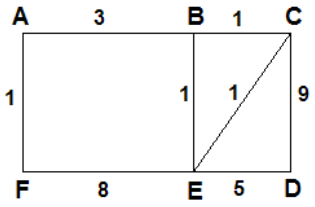
1. What is the difference between a spanning tree and a Hamiltonian circuit?
Both a spanning tree and a Hamiltonian circuit visit every vertex, but a Hamiltonian circuit must complete a closed circuit and end up where it started.

2. State one advantage and one disadvantage to a heuristic algorithm.
Advantage – It is fast. Disadvantage – It may not give an optimal solution

3. There are six juniors and six seniors on your math team including you. Each person lives at a different location. You want to plan to visit each of your teammates and return home. If it takes ½ minute to compute the total length of one tour, set up (but do not evaluate) the expression you would use to determine how long it will it take to apply the brute force algorithm to find the optimal tour.

$$\left(\frac{11!}{2}\right)\left(\frac{1}{2}\right)$$

4. It is possible to have more than one spanning tree with the same minimum cost in one graph. Find two different minimum-cost spanning trees for the graph below.




There are three possibilities, all with length 11. Students can give any two of the three possibilities.