

1. The base of a rectangle has a length of 8. One of the whole numbers greater than 5.2 and less than 7.3 is selected at random for the height of the rectangle. Find the probability that the area of this rectangle is greater than 50. Express your answer as a common fraction reduced to lowest terms.
2. If $f(x) = |3x - 12|$, then an equation of a line of symmetry is $x = k$. Find the value of k .
3. Give the smallest possible **positive** value of x for which $\cos(2x^\circ - 60^\circ) = 0$.
4. The cubic equation whose solution set is $\{1, 5, 11\}$ can be written as $x^3 + kx^2 + wx + p = 0$. Find the value of $(k + w + p)$.
5. The first term of a geometric sequence of real terms is 405. The fifth term of this sequence is 20480. Find the third term of this sequence.
6. A circle with center at $(-6, 3)$ contains the point $(2, -12)$. An arc of the circle has measure of $35^\circ 15'$. Find the area of the sector of the circle that is bounded by two radii and the arc. Express your answer as a **decimal** rounded to the nearest tenth.
7. One of the transformations necessary to produce the graph of $y = 2x^2 + 24x - 1$ from the graph of $y = x^2$ is a vertical shift k units downward. Find the value of $|k|$.
8. The function g satisfies the functional equation $g(x) + g(y) = g(x + y) + 2xy - 8$ for every pair (x, y) of real numbers. If $g(1) = 22$, find $g(-11)$.

9. When $y^2 + my + 14$ is divided by $(y - 3)$, the quotient is $f(y)$ and the remainder is k .
When $y^2 + my + 14$ is divided by $(y - 7)$, the quotient is $g(y)$ and the remainder is w .
If $w = 2k + 1$, find the value of m .
10. All lengths of the sides of a triangle are integers. The cosine of the smallest angle of the triangle is $\frac{19}{21}$. The length of one of the adjacent sides of the triangle for that angle is 6.
Find the smallest perimeter for such a triangle.
11. From the set $\{A, B, C, D, E, F\}$, two letters are selected at random without replacement.
Find the probability that A or B or both A and B were selected. Express your answer as a common fraction reduced to lowest terms.
12. If \ln is the symbol for natural logarithm, find the value of y such that
 $\ln(5 + 4y) - \ln(3 + y) = \ln(3)$.
13. In Triangle ABC , $AB = 64$, $BC = 64$, and $AC = 48$. A , B , and C are the centers of three mutually externally tangent circles whose points of tangency are D , E , and F .
Rounded to the nearest **whole number**, find the perimeter of Triangle DEF .
14. Last year UIC traced the job placement of their high school mathematics teacher candidates and found all took jobs teaching. The arithmetic mean salary of the candidates who took jobs in Chicago was \$48,600. The arithmetic mean of the rest of the candidates, who took jobs outside Chicago, was \$43,800. The overall arithmetic mean for all these candidates was \$45,000. Find the fraction of the candidates who took jobs outside Chicago. Express your answer as a common fraction reduced to lowest terms.

15. A farmer wishes to fence in a rectangular plot of ground bounded on one side by a river. He has 2200 feet of chain-link fence available. He will **not** fence the side along the river, but each of the two widths of the field perpendicular to the river will need a double layer (or “two-ply”) of fence. If the area of the rectangular plot enclosed is to be a maximum, find the number of feet in the length of the field that is parallel to the river. (Consider only the lengths of fencing used in the problem. Ignore the thickness of the double fencing in your computations.)
16. Find the absolute value in the difference of the two values of x for which the three terms $x-2$, $x+1$, and $4x-8$ (in that order) will form a geometric progression?
17. (Multiple Choice) Write A if the conditions below determine a conic. Write B if the conditions below overdetermine a conic (that is, there is no conic meeting all the given conditions). Write C if the conditions below underdetermine a conic (that is, there is more than one conic that meets all the given conditions).

Ellipse with center at $(0, 0)$ and eccentricity of $\frac{2}{3}$

Note: Be certain to write the correct capital letter as your answer.

18. If $\sin(3x^\circ) = \frac{1}{2}$ and $133^\circ < x < 178^\circ$, find the value of x .
19. The length of the latus rectum of a hyperbola is 24.12, the eccentricity of the hyperbola is 3.62, the center of the hyperbola is at $(8.16, 19.02)$, and the principal axis of the hyperbola is parallel to the y -axis. One of the foci of the hyperbola is located at (x, y) . Find the largest possible value of y . Express your answer as a **decimal** rounded to the nearest hundredth.
20. Find the **absolute** value of the distance from the point $(1, 4, 9)$ to the plane determined by the 3 points $(5, 0, 4)$, $(1, 7, 3)$, and $(2, 8, 0)$. Express your answer as a **decimal rounded to the nearest thousandth**.

1. $\frac{1}{2}$ (Must be this reduced common fraction.)
2. 4
3. 75 (Degrees optional.)
4. -1
5. 2880
6. 88.9 (Must be this decimal.)
7. 73
8. -470
9. -16
10. 16
11. $\frac{3}{5}$ (Must be this reduced common fraction.)
12. 4
13. 84 (Must be this whole number.)
14. $\frac{3}{4}$ (Must be this reduced common fraction.)
15. 1100 (Feet optional.)
16. 4
17. C (Must be this capital letter.)
18. 170 (Degrees optional.)
19. 22.63 (Must be this decimal.)
20. 1.028 (Must be this decimal.)