

1. From the set $\{5, 10, 15, \dots, 5n, \dots, 45\}$, one member is selected at random. Find the probability that the member selected is an **even** integer. Express your answer as a common fraction reduced to lowest terms.
2. Find the product of the five prime positive integral factors of 3570.
3. Find the sum of the first ten positive integers.
4. When $(2y + x)^5$ is expanded and completely simplified, what is the numerical coefficient of the y^2x^3 term?
5. Let $i = \sqrt{-1}$ and let k represent a real number. If $\frac{3-i}{k+4i} = 0.2 - 0.15i$, find the value of k .
6. The parabola which is the graph of the equation $y = ax^2 + bx + c$ passes through the points $(-16, -226)$, $(4, 64)$, and $(12, 208)$. Find the focus of this parabola. Express your answer as an **ordered pair** of the form (x, y) .
7. If the letters of "ILLINOIS" are jumbled, selected randomly one at a time without replacement, and written on a straight line in the order of the draw, find the probability that the three "I" letters are together, in adjacent positions in the random re-arrangement of letters. Express your answer as a common fraction reduced to lowest terms.
8. In a certain variation, z varies jointly as x and y and inversely as w^3 . In this variation, $z = 800$ when $x = 10$, $y = 64$, and $w = 2$. Find the value of z when $x = 5$, $y = 54$, and $w = 3$.
9. If the multiplicative inverse of $\begin{bmatrix} 1 & 3 \\ 2 & k \end{bmatrix}$ is $\begin{bmatrix} 4 & -1.5 \\ -1 & 0.5 \end{bmatrix}$, find the value of k .

10. The three real roots for x of $27x^3 + ax^2 + bx - 125 = 0$ are in geometric progression, and a and b are both integers. If the smallest root for x were decreased by $\frac{20}{9}$, then this resulting number and the other two roots for x would be in arithmetic progression. Find the value of a .
11. The sum of two numbers is 14. If $\frac{1}{3}$ of the square of the smaller is added to the larger, find the smallest possible result that can be obtained. Express your answer as an improper fraction reduced to lowest terms.
12. If the system $\begin{cases} 2x - 3y + z = 4 \\ 4x - 2y + 3z = 11 \\ x + 5y - 7z = 9 \end{cases}$ is solved by the determinant method for y , then:
- $$y = \frac{\begin{vmatrix} 2 & a & b \\ c & d & e \\ 1 & f & g \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 4 & -2 & 3 \\ 1 & 5 & -7 \end{vmatrix}}.$$
- Find the value of
- $(2c + 3d + 4g)$
- .
13. The lines f and k are symmetric to each other with respect to the line $y = 2x$. The equation of line f is $y = \frac{1}{8}x + \frac{1}{2}$. The equation of line k can be written as $y = mx + b$. Find the value of $(m + b)$. Express your answer as a decimal.
14. Let $i = \sqrt{-1}$. One of the three distinct cube roots of -8 can be expressed in the form $k + i\sqrt{w}$, where k and w are positive integers. Find the value of $(2k + 3w)$.

15. By substituting 1, 2, 3, and 4 for n in a quadratic expression for n , the first four terms are respectively 2, 6, 12, and 20. The $(n+1)^{\text{st}}$ term of this sequence is equal to $n^k + wn + p$ where k , w , and p are positive integers. Find the value of $(2k + 3w + 4p)$.
16. When completely simplified, one of the terms of the expansion of $(x + \frac{y}{2})^9$ is $\frac{k}{w}x^3y^p$ where k , w , and p are positive integers. Find the minimum value of $(k + w + p)$.
17. In a previous year, 8412 students participated in the ICTM State Math Contest. Let each of those students receive a number, and let the numbers be 8412 consecutive integers starting with 1 and ending with 8412. If 5 of those numbers were drawn at random (one at a time without replacement), find the probability that the 5 numbers drawn were in descending order. Express your answer as a common fraction reduced to lowest terms.
18. How many distinct integral multiples of 6 are there between 1 and 207?
19. The terms of an arithmetic sequence are 175, 185, 195, \dots . The terms of the triangular sequence whose n^{th} term is $\frac{n^2 + n}{2}$ are 1, 3, 6, 10, \dots . Find the value of the first term of the triangular sequence that is greater than the corresponding term of the arithmetic sequence.
20. By substituting 1, 2, 3, 4, 5, and 6 for x in a polynomial expression for x , the first six terms are respectively 2, 24, 72, 158, 294, and 492. If $P(x)$ is the polynomial expression of lowest degree satisfying the given, find $P(37)$.

2010 SA

Name _____ **ANSWERS**

Algebra II

School _____

(Use full school name – no abbreviations)

_____ Correct X **2** pts. ea. =

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. _____ $\frac{4}{9}$ (Must be this reduced common fraction.)

11. _____ $\frac{53}{4}$ (Must be this reduced improper fraction.)

2. _____ 3570 _____

12. _____ 13 _____
-0.75 OR -.75

3. _____ 55 _____

4. _____ 40 _____

14. _____ 11 _____

5. _____ 12 _____

15. _____ 21 _____

6. _____ $(-64, -512)$ (Must be this ordered pair.)

16. _____ 43 _____

7. _____ $\frac{3}{28}$ (Must be this reduced common fraction.)

17. _____ $\frac{1}{120}$ (Must be this reduced common fraction.)

8. _____ 100 _____

18. _____ 34 _____

9. _____ 8 _____

19. _____ 496 _____

10. _____ -195 _____

20. _____ 102854 _____

ITEM ANALYSIS	
Div 1A – 140 papers	
% correct	
1. 73%	11. 5%
2. 54%	12. 6%
3. 82%	13. 1%
4. 44%	14. 9%
5. 22%	15. 5%
6. 7%	16. 20%
7. 7%	17. 2%
8. 32%	18. 79%
9. 29%	19. 19%
10. 2%	20. 6%

ITEM ANALYSIS	
Div 2A – 145 papers	
% correct	
1. 91%	11. 15%
2. 68%	12. 9%
3. 89%	13. 2%
4. 70%	14. 15%
5. 55%	15. 18%
6. 13%	16. 43%
7. 6%	17. 2%
8. 37%	18. 81%
9. 64%	19. 41%
10. 3%	20. 26%