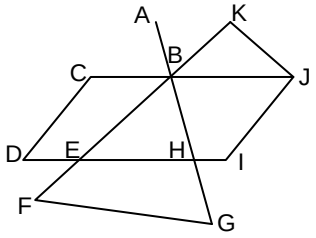
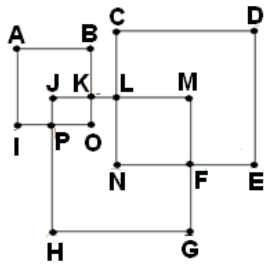
Questions for the Oral Competition – Division A, State Finals 2010

1. a) Explain the difference between an Euler circuit and a Hamiltonian circuit.
   b) Determine whether each of the figures below is an Euler circuit and/or a Hamiltonian circuit.

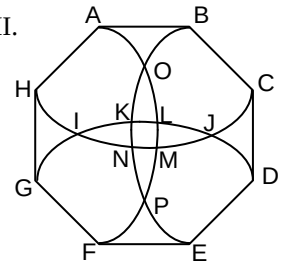
I.



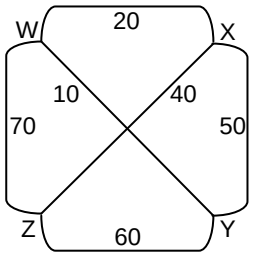
II.



III.



2. Use the algorithms listed to solve the traveling salesman problem with the given graph below. Start at vertex W. Explain the procedures that you used for each method.
   a. brute force
   b. nearest neighbor
   c. sorted edges



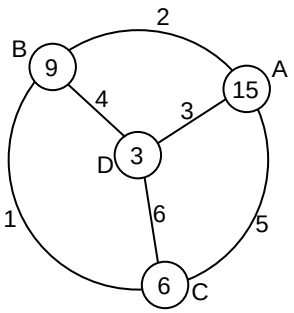
3. A manufacturing process consists of 6 tasks, each of which takes the given amount of time:

Task A:	4 minutes
Task B:	6 minutes
Task C:	4 minutes
Task D:	5 minutes
Task E:	6 minutes
Task F:	3 minutes

   Task A must be completed before either Task B or C can be started. Tasks A and B must be completed before Task D can be started**.** Tasks A and C must be completed before Task E can be started. Task F must be completed before Task C can be started.

   a. Draw an order requirement diagraph for this manufacturing process
   b. Find the critical path that gives the earliest completion time for the process.

4. In the graph below, the number in the circle at each vertex is the cost of relaying a message through that vertex. The number on an edge indicates the cost of providing service between the endpoints of the edge. A relay can send a message to only one vertex at a time. There is no cost to initiate or receive a message at a vertex, only to relay the message through the vertex. Begin at vertex A and find the minimum cost for sending a message to all vertices, including the relay costs.

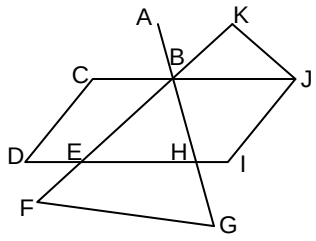


Solutions for the Oral Competition – Division A, State Finals 2010

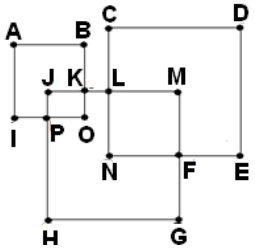
1. a. Explain the difference between an Euler circuit and a Hamiltonian circuit.
An Euler circuit must cover each edge only once and return to its starting location. A Hamiltonian circuit must visit each vertex only once and return to its starting location.

b. Determine whether each of the figures below is an Euler circuit and/or a Hamiltonian circuit.

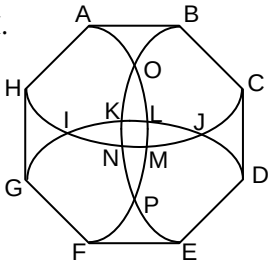
I.



II.

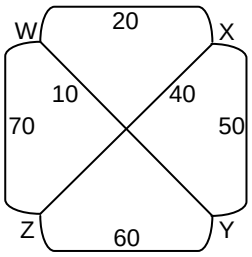


III.



I) No Euler, No Hamiltonian II) Yes Euler, No Hamiltonian III) No Euler, Yes Hamiltonian

2. Use the algorithms listed to solve the traveling salesman problem with the given graph below. Start at vertex W. Explain the procedures that you used for each method.
a. brute force
b. nearest neighbor
c. sorted edges



a. WXYZW = 20 + 50 + 60 + 70 = 200
WXZYW = 20 + 40 + 60 + 10 = 130 (optimal)
WYXZW = 10 + 50 + 40 + 70 = 170

b. At each vertex, choose least cost. Therefore, WYXZW = 170

c. Sort edges from least to most: WY = 10, WX = 20, XZ = 40 plus connector ZY = 60 for total circuit WXZYW = 130

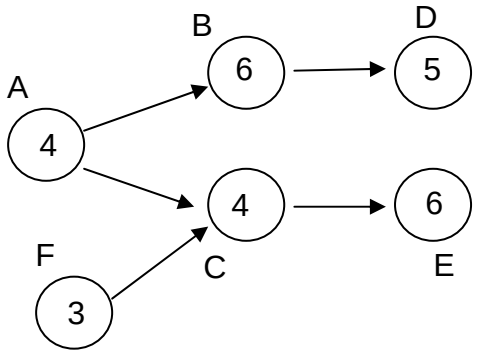
3. A manufacturing process consists of 6 tasks, each of which takes the given amount of time:

Task A: 4 minutes
Task B: 6 minutes
Task C: 4 minutes
Task D: 5 minutes
Task E: 6 minutes
Task F: 3 minutes

Task A must be completed before either Task B or C can be started. Tasks A and B must be completed before Task D can be started**.** Tasks A and C must be completed before Task E can be started. Task F must be completed before Task C can be started.

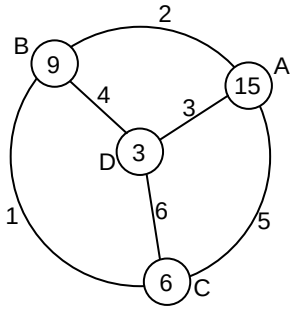
a. Draw an order requirement diagraph for this manufacturing process
b. Find the critical path that gives the earliest completion time for the process.

a. The order requirement diagraph is:



b. The possible paths are ABD with length 15, ACE with length 14 and FCE with length 13. The critical path is FCE with completion time 13 minutes.

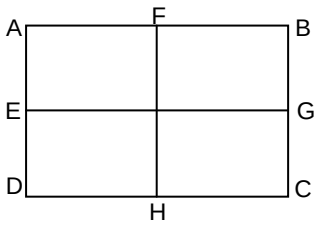
4. In the graph below, the number in the circle at each vertex is the cost of relaying a message through that vertex. The number on an edge indicates the cost of providing service between the endpoints of the edge. A relay can send a message to only one vertex at a time. There is no cost to initiate or receive a message at a vertex, only to relay the message through the vertex. Begin at vertex A and find the minimum cost for sending a message to all vertices, including the relay costs.



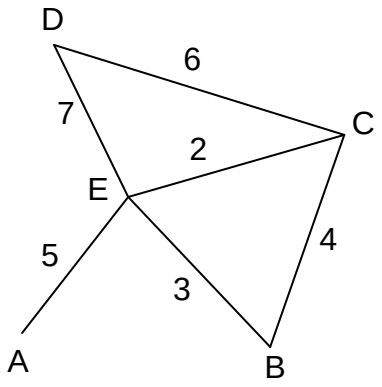
ABCD = 2 + 9 + 1 + 6 + 6 = 24
ABDC = 2 + 9 + 4 + 3 + 6 = 24
ADCB = 3 + 3 + 6 + 6 + 1 = 19 (this is the least cost)
ADBC = 3 + 3 + 4 + 9 + 1 = 20
ACDB = 5 + 6 + 6 + 3 + 4 = 24
ACBD = 5 + 6 + 1 + 9 + 4 = 25

Extemporaneous Questions for the Oral Competition – Division A, State Finals 2010

1. Define Eulerization and find a best Eulerization for the graph below.



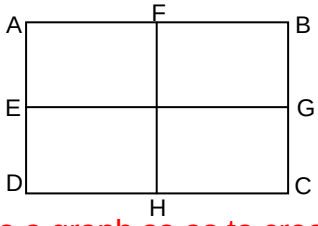
2. Use Kruskal's algorithm to find the minimum cost spanning tree for the graph below:



3. Suppose it takes 0.5 minutes to generate one Hamiltonian circuit tour of cities. What is the maximum number of cities on the tour that you can find in 1 day (1440 minutes) by using the brute force algorithm?

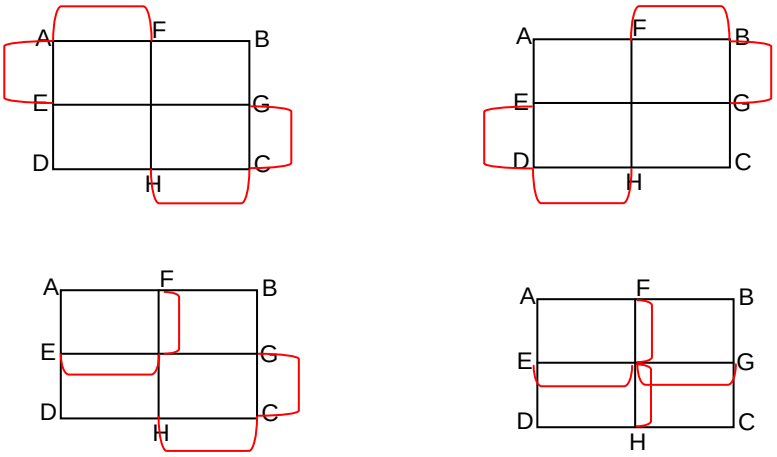
Extemporaneous Solutions for the Oral Competition – Division A, State Finals 2010

1. Define Eulerization and find a best Eulerization for the graph below.

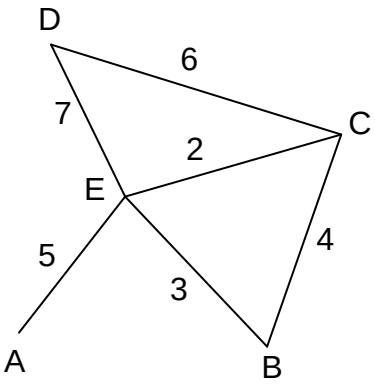


Eularization – adding new edges to a graph so as to create a graph that possesses an Euler circuit.

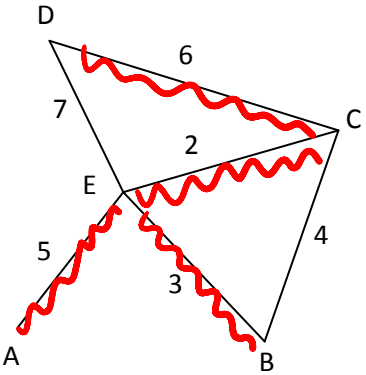
Some possibilities are:



2. Use Kruskal's algorithm to find the minimum cost spanning tree for the graph below:



Include edges in order so that no circuits are formed. Edges included would be:
EC (2), EB (3), AE (5) and CD (6) for minimum cost 16.



3. Suppose it takes 0.5 minutes to generate one Hamiltonian circuit tour of cities. What is the maximum number of cities on the tour that you can find in 1 day (1440 minutes) by using the brute force algorithm?

$$\frac{(n-1)!}{2}(0.5) \le 1440$$

$$(n-1)! \le 5760$$

$$n = 8$$