

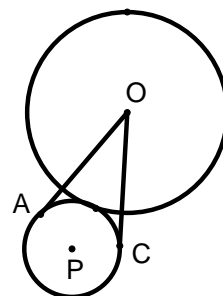
1. In quadrilateral $ABCD$, $\angle ABC = 102^\circ$. From the set of integral degree values $\{1^\circ, 2^\circ, 3^\circ, \dots, n^\circ, \dots, 84^\circ\}$ one member is selected at random as the measure of $\angle DAB$. Find the probability that $\angle ABC$ is supplementary to $\angle DAB$. Express your answer as a common fraction reduced to lowest terms.
2. Let vector $\vec{c} = (1, -4, k)$ and let vector $\vec{d} = (4, 6, 1)$. If the two given vectors are perpendicular to each other, find the value of k .
3. For all positive integers n such that $n > 1$, $t_n = 2t_{(n-1)}$. If $t_1 = 3$, find the value of t_3 .
4. Find the sum of the next four terms after 13 in the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, ...
5. If $\tan(x^\circ) = \frac{1}{3}$, and if $x - y = 45^\circ$, find $\tan(y^\circ)$. Express your answer as a common fraction reduced to lowest terms.
6. Let $a_n = \frac{2n}{n^2 + 2}$. Let $L = \lim_{n \rightarrow \infty} \left(\frac{2n}{n^2 + 2} \right)$. Find the smallest **positive** integral value of n for which $|L - a_n| < \frac{1}{12}$.
7. The equation of a parabola whose vertex is at $(0, -2)$ and whose focus is $(0, 11)$ can be written as $x^2 = k(y + w)$. Find the value of $(k + w)$.
8. If a normal, fair cubical die is rolled 7 times, find the probability that the uppermost face will be either a 2 or a 4 exactly 4 times, either a 1, 3 or 5 exactly twice, and a 6 exactly once. Express your answer as a decimal rounded to 4 significant digits.

9. The coordinate axes of the graph of the equation $7x^2 + 6\sqrt{7}xy - 6y^2 = 35$ are rotated through a positive angle (θ) such that $\theta > 180^\circ$ so as to eliminate the xy term. Find the smallest possible value of θ . Express the degree measure of θ as a **decimal** rounded to the nearest hundredth of a degree.
10. The graph of $y = \frac{12x^2 + 21x - 2}{2x^2 - x - 8}$ has a horizontal asymptote of $y = k$. Find the value of k .
11. A rectangle that is **not** a square has both diagonals drawn. From the four sides and the two diagonals, two segments are selected at random without replacement. Find the probability that the two segments are congruent. Express your answer as a common fraction reduced to lowest terms.
12. Let k represent a positive integer. If $\frac{k\pi}{12}$ is a radian measure, find the smallest possible value of k such that $\tan\left(\frac{k\pi}{12}\right) = 2 + \sqrt{3}$.
13. Tiger tees off on the 16th hole of the Illini golf course. His ball leaves the tee (assume at ground level) with an initial velocity of 186.4 feet per second and at an initial angle with the horizontal of 28.46° . Assuming the ground is level and that the effect-on-gravity vector is $(0, -16t^2)$ where t is measured in seconds, find the horizontal distance in feet that the ball will have traveled when it first hits the ground after launch. Express your answer as a **decimal** rounded to 4 significant digits.
14. $S = \{3100, 4000, 3693, 2800, 2100, 1300, 4700, 3000, 2595, 1302, 4200, 5400, 2605, 2900, 2400\}$
Set S contains 15 distinct elements. The six-number summary—that is, the minimum data value; the first quartile; the median; the third quartile; the maximum data value; and the arithmetic mean are respectively 1300, 2400, 2900, 4000, 5400, and 3073
- Let x be an integral multiple of 1001 such that $1000 < x < 9999$. Let a second set consist of the 15 distinct elements of the above given set S , and let the sixteenth element of this second set be x . If at least 5 values of the six-number summary of the second set are different than their respective values of the six-number summary of the original 15 element set, find the sum of all possible distinct values of x .

15. In a room in a shape of a rectangular solid with a high ceiling, a cube with an edge of 3 is placed so that one face is flat against a wall and an adjacent face is flat against the floor. A ladder with a length of 16 is placed in such a way that the top is flat against the wall and the ladder rests completely against the free horizontal edge of the cube. Find the maximum number of units in the height above the floor that is touched by the top of the ladder. Express your answer as a **decimal** rounded to the nearest hundredth.
16. The first term of a geometric sequence is a negative number, the second term is 2, and the fourth term is 4. Find the third term.
17. Point A has a coordinate of 0 and Point E has a coordinate of 100 on line segment \overline{AE} . B and C are two points between A and E with C between B and E . Let point B be chosen at random such that $BC = 10$. A point D is picked at random between A and E . If the probability that the coordinate of D is at least k more than the coordinate of B is 0.4675, find the value of k . Express your answer as an **exact decimal**.

18. One of the vertical asymptotes of the graph of $f(x) = \frac{x-7}{x^2-5x+6}$ is $x=3$. The other vertical asymptote is $x=k$. Find the value of k .

19. In the diagram, a circle with center at P is externally tangent to the circle with center at O . Circle P is also tangent to \overline{OA} at A and to \overline{OC} at C . $\angle AOC = 28^\circ$, and the length of a radius of the circle with center at P is 9.438. Find the length of a radius of the circle with center at O . Express your answer as a **decimal** rounded to the nearest hundredth.



20. The Polar Coordinates of points A , B , and C are $A(12, 49^\circ)$, $B(77, 77^\circ)$, and $C(48, 2^\circ)$. Find the radius of the **inscribed** circle of $\triangle ABC$. Round your answer for the radius to the nearest integer and express your answer as that **integer**.

2010 SA

Pre-Calculus

Name ANSWERS

School _____

(Use full school name – no abbreviations)

_____ Correct X 2 pts. ea. =

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. _____ $\frac{1}{84}$ (Must be this reduced common fraction.)

11. _____ $\frac{1}{5}$ (Must be this reduced common fraction.)

2. _____ 20 _____

12. _____ 5 _____
(Must be this decimal.)

3. _____ 12 _____

13. 909.8 _____

4. _____ 199 _____

14. _____ 31031 _____

5. _____ $-\frac{1}{2}$ OR $\frac{-1}{2}$ OR $\frac{1}{-2}$ (Must be this reduced common fraction.)

15. _____ 15.56 (Must be this decimal.)

6. _____ 24 _____

16. _____ $-2\sqrt{2}$ (Must be this exact answer.)

7. _____ 54 _____

17. _____ 8.25 (Must be this decimal.)

8. _____ 0.05401 OR _____ (Must be this decimal.)
_____ .05401 _____

18. _____ 2 _____

9. _____ 205.34 (Must be this decimal.)

19. _____ 29.57 (Must be this decimal.)

10. _____ 6 _____

20. _____ 15 (Must be this integer.)

ITEM ANALYSIS	
Div 1A – 111 papers	
% correct	
1. 68%	11. 23%
2. 13%	12. 74%
3. 64%	13. 3%
4. 73%	14. 7%
5. 54%	15. 1%
6. 24%	16. 31%
7. 11%	17. 1%
8. 3%	18. 89%
9. 0%	19. 15%
10. 68%	20. 4%

ITEM ANALYSIS	
Div 2A – 134 papers	
% correct	
1. 90%	11. 42%
2. 29%	12. 78%
3. 82%	13. 8%
4. 81%	14. 17%
5. 68%	15. 2%
6. 42%	16. 52%
7. 15%	17. 1%
8. 3%	18. 95%
9. 1%	19. 21%
10. 78%	20. 8%