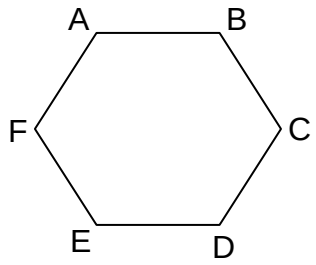
Questions for the Oral Competition – Division AA, State Finals 2010

1) Using only polygons with 15 or fewer sides, can a semiregular tiling exist whose vertex figure contains exactly one octagon?

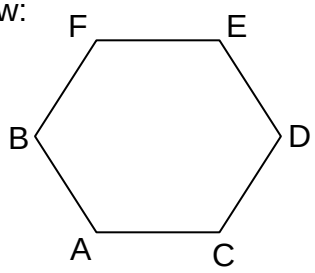
2) What combination(s) of regular shapes that contain at least one regular hexagon can be used to tile a plane?

3) Does the set of odd integers form a group under the operation of addition?

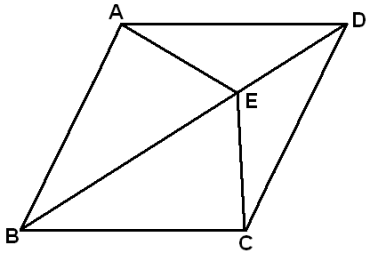
4) Consider the regular hexagon ABCDEF and the rigid motion of a
- a reflection H across a horizontal line of symmetry of the hexagon
- a reflection V across a vertical line of symmetry of the hexagon
- a rotation R of $60°$ about the center of the hexagon



Explain why any combination of these rigid motions cannot result in the hexagon shown below:



5) Below is a drawing of a Penrose tile. If the length of $\overline{DE}$ is 2 units, give the exact lengths of the segments $\overline{BE}$, $\overline{AE}$, $\overline{CE}$, $\overline{AB}$, $\overline{AD}$, $\overline{DC}$ and $\overline{BC}$.



Questions for the Oral Competition – Division AA, State Finals 2010

SOLUTIONS

1) Using only polygons with 15 or fewer sides, can a semiregular tiling exist whose vertex figure contains exactly one octagon?

No. One octagon has an interior angle of $135°$ and no other combination of regular figures with less than 20 sides have interior angles which would allow the angles around each vertex to add to $360°$

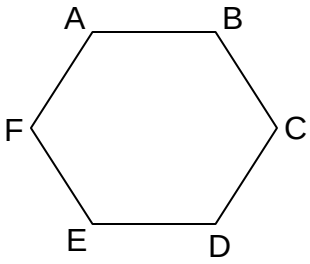
2) What combination(s) of regular shapes that contain at least one regular hexagon can be used to tile a plane?

3 hexagons
2 hexagon, 2 triangles
1 hexagon, 4 triangles
1 hexagon, 1 triangle, 2 squares

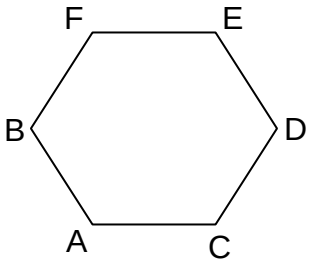
3) Does the set of odd integers form a group under the operation of addition?

No. Closure does not hold (odd integer + odd integer = even integer)

4) Consider the regular hexagon ABCDEF and the rigid motions of a
- a reflection H across a horizontal line of symmetry of the hexagon
- a reflection V across a vertical line of symmetry of the hexagon
- a rotation R of $60°$ about the center of the hexagon

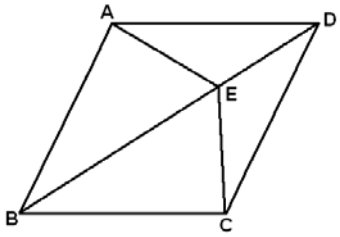


Explain why any combination of these rigid motions cannot result in the hexagon shown below:

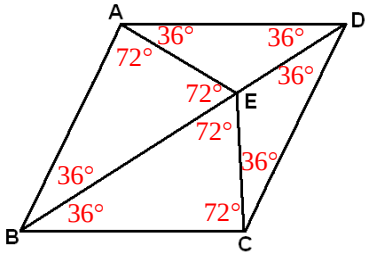


All of the rigid motions maintain size, shape, and order of the vertices. Since the vertices of the resulting figure (BACDEF) are not in the same order as the original figure (ABCDEF), it is not possible.

5) Below is a drawing of a Penrose tile. If the length of $\overline{DE}$ is 2 units, give the exact lengths of the segments $\overline{BE}$, $\overline{AE}$, $\overline{CE}$, $\overline{AB}$, $\overline{AD}$, $\overline{DC}$ and $\overline{BC}$.



The ratio of the segments of the diagonal in a Penrose tile is $1:\phi$ (shorter segment: longer segment). If DE=2, then BE= 2ϕ. With angles as shown below, BE=BC=AB and DE=EC=AE. Since the Penrose tile is a rhombus, AD=CD=AB=BC .

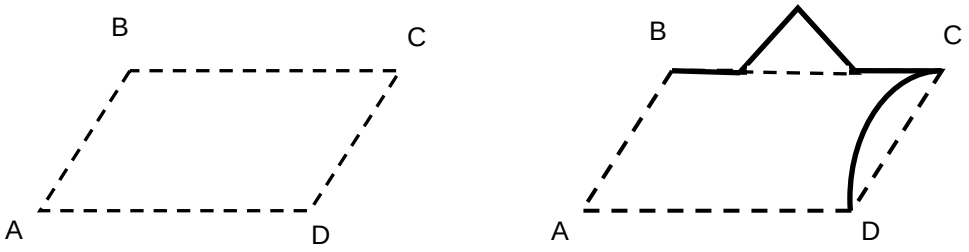


DE = EC = AE = 2
BE = AB = AD = CD = BC = 2ϕ

Extemporaneous Questions for the Oral Competition – Division AA, State Finals 2010

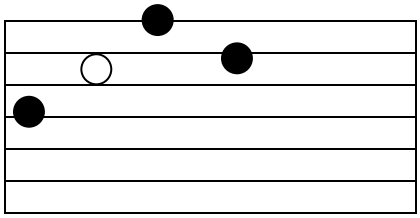
1) Which regular polygons can be used to create a monhedral tiling?

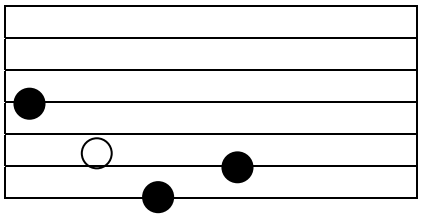
2) The parallelogram ABCD (with dashed lines) is being adapted as shown by the solid lines in the second figure to be used to tile the plane. Describe how segments $\overline{AB}$ and $\overline{AD}$ in the second figure need to be changed to be able to do so.



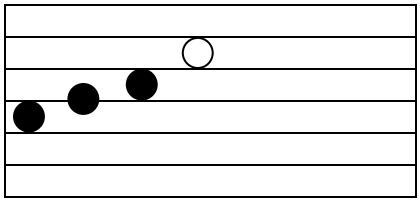
3) Geometric transformations also occur in musical competitions. For each piece of "music" shown below on a unique ICTM music staff with unique "notes," state whether the second figure illustrates a reflection, rotation, translation, or glide reflection of the first. Explain each answer.

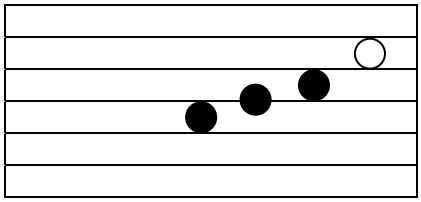
a)



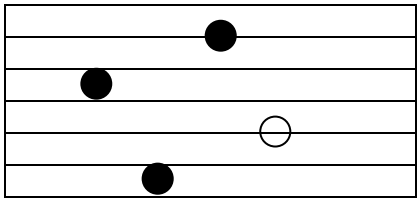


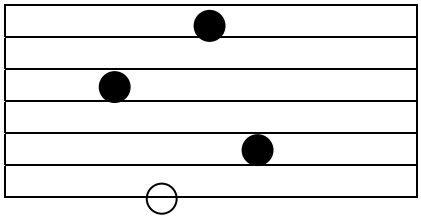
b)



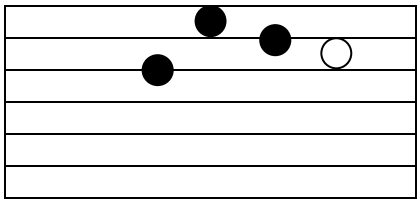


c)





4) If a glide reflection is performed on the following group of "notes," sketch the resulting figure.



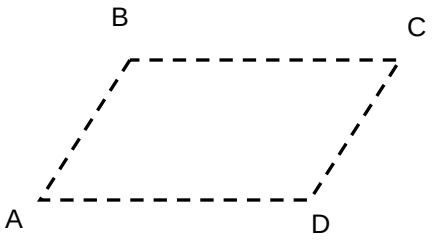
Extemporaneous Questions for the Oral Competition – Division AA, State Finals 2010

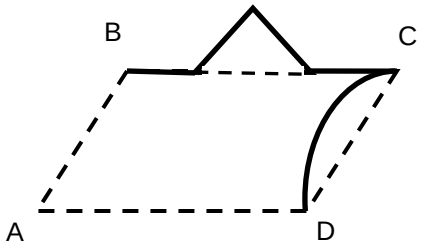
SOLUTIONS

1) Which regular polygons can be used to create a monhedral tiling?

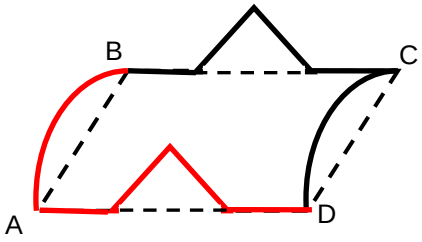
Triangles, squares and hexagons are the only regular figures that can create a monhedral tiling.

2) The parallelogram ABCD (with dashed lines) is being adapted as shown by the solid lines in the second figure to be used to tile the plane. Describe how segments $\overline{AB}$ and $\overline{AD}$ in the second figure need to be changed to be able to do so.



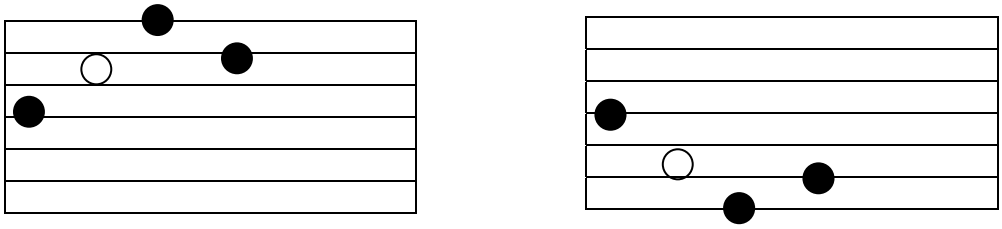


Solution:



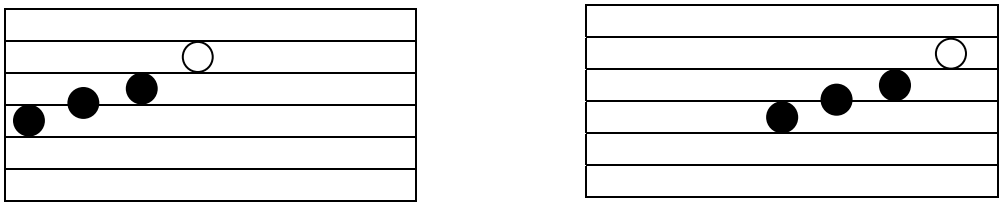
3) Geometric transformations also occur in musical competitions. For each piece of "music" shown below on a unique ICTM music "staff" with unique "notes," state whether the second figure illustrates a reflection, rotation, translation, or glide reflection of the first. Explain each answer.

a)



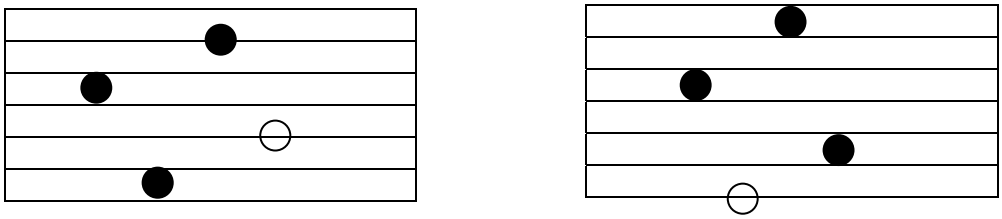
a) reflection – the second set of notes has been reflected over the middle line in the staff.

b)



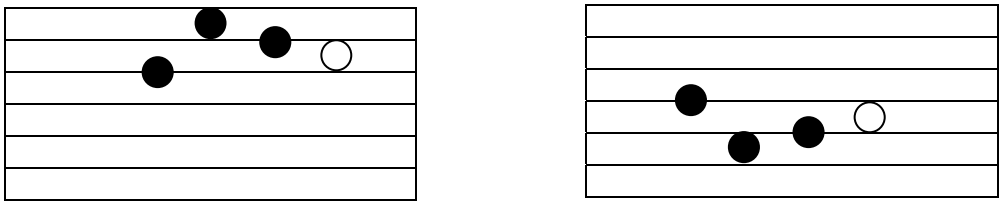
b) translation – the second set of notes has been shifted to the right along a horizontal line

c)



c) rotation – the second set of notes has been rotated 90 degrees to the right.

4) Perform a glide reflection on the following group of "notes" and sketch the resulting figure.



One possibility is shown here – credit should be given for any valid glide reflection.