

1. Let  $k$  be selected at random from the set  $\{1, 2, 3, 4, 6, 8, 12, 18\}$ . If  $3x = k$ , find the probability that the solution for  $x$  is a whole number. Express your answer as a common fraction reduced to lowest terms.
2. Let  $x$  be a positive integer such that  $x < 100$ . For how many distinct values of  $x$  is  $\sin(2x) + 1 > 2$ ?
3. If the sides of a triangle have lengths of  $3a$ ,  $2b$ , and  $c$ , with  $\angle A$  opposite side of length  $3a$ , then  $pa^2 = qb^2 + rc^2 - cwb \cos(A)$ . If  $p$ ,  $q$ ,  $r$ , and  $w$  are all positive integers, find the smallest possible value of  $(p + q + r + w)$ .
4. If  $f(x) = 3x - 4$ , then an equation for the inverse can be written as  $f^{-1}(x) = \frac{x+k}{w}$ . Find the value of  $(k - w)$ .
5. If  $x$  is a positive integer, find the smallest possible value of  $x$  such that  $\cot(x^\circ)$  and  $\cot(-x^\circ)$  are equal numbers.
6. From physics, the height  $s$ , in feet, of an object after  $t$  seconds that has been thrown straight up from a point  $s_0$  feet above ground level with an initial velocity of  $v_0$  is  $s = -16t^2 + v_0t + s_0$ . If a baseball is thrown straight up from 3 feet above ground level with initial velocity of 97 feet per second, find the number of feet in the maximum height that the ball will reach. Express your answer as a **decimal** rounded to the nearest hundredth of a foot.
7. A right circular cylinder has a volume of  $12\pi$ . The lengths of the radius ( $r$ ) of the base and the height ( $h$ ) of the cylinder are both positive integers. Find the sum of all possible distinct values of  $(r + h)$  where  $r$  and  $h$  represent the radius and height of one cylinder.
8. The probability that Mary will have one of the top ten Precalculus scores is 0.4263. The probability that Peter will have one of the top ten Precalculus scores is 0.3642. The probability that both will have scores among the top ten Precalculus scores is 0.1968. Expressed as a **decimal** rounded to 4 significant digits, find the probability that both had scores in the top ten Precalculus scores if it is known that at least one of the two did.
9. Rounded to the nearest year, find the number of years needed for a sum of money to quadruple if invested at 7% annual percentage rate compounded continuously.
10. In  $\triangle ABC$ ,  $AB = 7$ , and  $AC = 8$ .  $\cos(\angle BAC)$  is a rational number that can be expressed as  $\frac{k}{w}$  where  $k$  and  $w$  are relatively prime positive integers. It is known that  $(k + w)$  is the square of a positive integer and that  $BC$  is a positive integer. Find the sum of all possible distinct values of  $BC$ .

11. A box that contains exactly 35 yellow marbles, 7 green marbles, and 8 red marbles. From this box, 5 marbles are drawn (without replacement) at random. Find the probability that the 5 marbles drawn were exactly 3 yellow, 1 green, and 1 red. Express your answer as a common fraction reduced to lowest terms.
12. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice. Given: A parabola whose points are all equidistant from the point  $(2, 5)$  and the line whose equation is  $x = 7$  and that has a latus rectum of length  $16\sqrt{5}$ .
- A) The conditions above determine a conic.  
 B) The conditions above overdetermine a conic (that is, there is no conic meeting all the given conditions).  
 C) The conditions above underdetermine a conic (that is, there is more than one conic that meets all the given conditions).

**Note: Be certain to write the correct capital letter as your answer.**

13. On the circle whose equation is  $(x - 25)^2 + (y - 30)^2 = 145$  a particle is at  $A(26, 42)$  and another particle is at  $B(24, 18)$ . At the same instant, both particles begin to move on the circle. The particle at  $A$  moves at a constant speed counter-clockwise on the circle, and the particle at  $B$  moves at a constant speed clockwise on the circle. The two particles meet for the first time at  $(17, 39)$ . The particle that started at  $A$  is moving  $k$  times the rate of the particle that started at  $B$ . Find the value of  $k$ . Express your answer as a **decimal** rounded to the nearest hundredth.
14. In this problem, assume that the standard deviation is calculated according to the standard method of calculating the standard deviation for a set of sample proportions. Also, assume the following table of  $z$ -scores with the accompanying standard normal probabilities is accurate. On the table  $-2(.0228)$  means that there is a normal probability of .0228 of obtaining a  $z$ -score of  $-2$  or less.

$-2(.0228)$	$-1.9(.0287)$	$-1.8(.0359)$	$-1.7(.0446)$	$-1.6(.0548)$
$-1.5(.0668)$	$-1.45(.0735)$	$-1.4(.0808)$	$-1.35(.0885)$	$-1.3(.0968)$
$-1.25(.1056)$	$-1.2(.1151)$	$-1.1(.1357)$	$-1(.1587)$	$-0.9(.1841)$
$-0.8(.2119)$	$-0.75(.2266)$	$-0.7(.2420)$	$-0.6(.2743)$	$-0.5(.3085)$
$-0.4(.3446)$	$-0.3(.3821)$	$-0.25(.4013)$	$-0.1(.4602)$	$0(.5000)$
$0.1(.5398)$	$0.2(.5793)$	$0.25(.5987)$	$0.3(.6179)$	$0.4(.6554)$
$0.5(.6915)$	$0.6(.7257)$	$0.7(.7580)$	$0.75(.7734)$	$0.8(.7881)$
$0.9(.8159)$	$1.0(.8413)$	$1.1(.8643)$	$1.2(.8849)$	$1.25(.8944)$
$1.3(.9032)$	$1.4(.9192)$	$1.5(.9332)$	$1.6(.9452)$	$1.7(.9554)$
$1.75(.9599)$	$1.8(.9641)$	$1.9(.9713)$	$2.0(.9772)$	$2.1(.9821)$
$2.2(.9861)$	$2.25(.9878)$	$2.3(.9893)$	$2.4(.9918)$	$2.5(.9938)$

Assume that 80% of all mathletes know the value of  $\sqrt{3}$  to 4 significant digits. In a sample observation, Carol selects 100 mathletes at random. Find the probability that the proportion of those mathletes who know the value of  $\sqrt{3}$  to 4 significant digits is between 76% and 82%. Express your answer as a **decimal** rounded to 4 decimal places.

15. Let  $a$ ,  $b$ , and  $c$  be positive integers. Find all **ordered triples**  $(a, b, c)$  such that

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & 1 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 148 \\ 157 \end{bmatrix}. \text{ Be certain to express your answer as } \mathbf{ordered\ triples} \text{ of the form } (a, b, c).$$

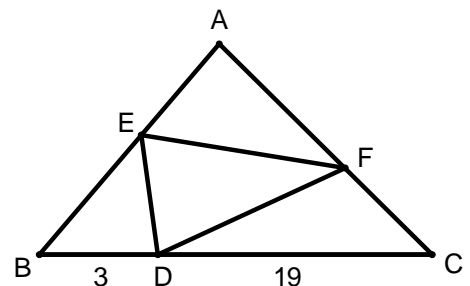
16. If  $x^4 - 15x^2 + 36$  is factored over the reals into four linear factors, one of those linear factors, in simplest form, is  $x - k\sqrt{w}$  where  $k$  and  $w$  are positive integers greater than one. Find the value of  $(5k + 3w)$ .

17. If a stick is broken at random into 3 pieces, find the probability that the longest of the 3 pieces is at least three times the shortest of the 3 pieces and that the 3 pieces can serve as the 3 sides of a triangle. Express your answer as a common fraction reduced to lowest terms.

18. If  $f(x) = \begin{cases} x-2 & \text{if } x \leq 0 \\ x^2+1 & \text{if } x > 0 \end{cases}$ , then the range of  $f$  can be expressed in interval notation as  $(-\infty, k] \cup (w, \infty)$ . Find the value of  $(2k + 11w)$ .

19. The coordinate axes of the graph of the equation  $7x^2 + 8\sqrt{6}xy - 5y^2 - 28 = 0$  are rotated through a positive acute angle  $(\theta)$  so as to eliminate the  $xy$  term. Find the value of  $\sin(\theta)$ . Express your answer as a **decimal** rounded to the nearest ten-thousandth.

20. In the figure shown,  $AB = AC = 15$ ,  $BD = 3$ , and  $DC = 19$ . Points  $A$ ,  $E$ , and  $B$  are collinear; points  $B$ ,  $D$ , and  $C$  are collinear; and points  $A$ ,  $F$ , and  $C$  are collinear.  $AE = DE$ , and  $AF = DF$ . Find  $DE$ . Express your answer as a **decimal** rounded to the nearest hundredth.



# 2010 SAA

Name ANSWERS

## Pre-Calculus

School \_\_\_\_\_

(Use full school name – no abbreviations)

Correct X 2 pts. ea. = 

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1.  $\frac{1}{2}$  (Must be this reduced common fraction.)

2. 0 OR "Zero" OR "None"

3. 18

4. 1

5. 90 (Degrees optional.)

6. 150.02 (Must be this decimal.)

7. 18

8. 0.3315 OR .3315 (Must be this decimal.)

9. 20 (Years optional.)

10. 14

11.  $\frac{187}{1081}$  (Must be this reduced common fraction.)

12. B (Must be this capital letter.)

13. 0.35 OR .35

14. 0.5328 OR .5328

15. (1,3,19), (6,1,18) (Must be these ordered triples, in either order.)

16. 19

17.  $\frac{1}{14}$  (Must be this reduced common fraction.)

18. 7

19. 0.4888 OR .4888 (Must be this decimal.)

20. No solution

## ITEM ANALYSIS

177 Papers

% correct

## 3AA Pre-Calculus

1. 91% 11. 22%

2. 82% 12. 57%

3. 33% 13. 10%

4. 85% 14. 14%

5. 53% 15. 14%

6. 69% 16. 63%

7. 71% 17. 1%

8. 5% 18. 77%

9. 66% 19. 3%

10. 14% 20. 0%

159 Papers

(Must be this decimal.)

% correct

## 4AA Pre-Calculus

1. 93% 11. 38%

2. 89% 12. 72%

(Must be this decimal.)

3. 56% 13. 17%

4. 92% 14. 15%

5. 67% 15. 29%

6. 83% 16. 78%

(Must be these ordered triples, in either order.)

7. 73% 17. 0%

8. 15% 18. 85%

9. 75% 19. 2%

10. 23% 20. 0%