NO CALCULATORS

1. If $1.2x = 30$, find the value of x.

2. The expression $2\sqrt{27} + 5\sqrt{\frac{4}{3}}$ can be simplified to a single term $\frac{k\sqrt{w}}{p}$ where k, w, and p are positive integers. Find the least possible value of $(k + w + p)$.

3. For all values of x, $\frac{2x+5}{4} + \frac{3x-4}{5}$ can be simplified to a single fraction $\frac{kx+w}{p}$. Find the least positive value of $(k + w + p)$.

4. A triangle with sides of lengths 42, 144, and 150 is inscribed in a circle whose circumference is $k\pi$. Find the value of k.

5. b is a y-intercept of a relation if the point $(0, b)$ lies on the graph of the relation. Find the sum of all possible distinct y-intercept(s) of $(y-5)^2 = x + 9$.

6. In a circle whose equation is $x^2 + y^2 + 10x - 14y - 26 = 0$, $(-11, 15)$ is one endpoint of a diameter of the circle. Find the coordinates of the other endpoint of this diameter.

7. **(Always, Sometimes, or Never)** For your answer, write <u>*the whole word* ***Always***</u>**,** **Sometimes,** or **Never**—whichever is correct.

   If a sphere is inscribed in a cube, then the ratio of the surface area of the sphere to the surface area of the cube is the same as the ratio of the length of a diameter of the sphere to the length of an edge of the cube.

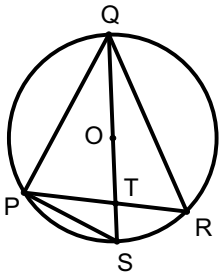
8. When expressed in base fifty-six, $N!$ terminates in a block of exactly 11 zeroes. Compute the largest positive integer N with this property. Express your answer for N in base ten notation.

NO CALCULATORS

9. In simplest radical form, $\frac{2}{\sqrt[3]{243}} = \frac{k\sqrt[3]{w}}{p}$, where k, w, and p are positive integers. Find the least possible value of $(k+w+p)$.

10. Find the value of $(168542)^2 - (168539)^2$.

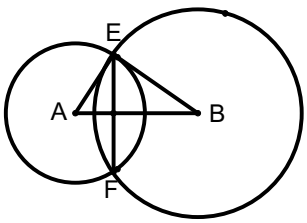
11. Given: P, Q, R, and S lie on a circle with a center at O. Q, O, T, and S lie on a diameter. $\overset{\frown}{PQ} = 112°$, $\angle RPS = 27°$, points P, T, and R are collinear. Find the degree measure of $\angle PQR$.



12. A woman working alone and at a constant rate can do a job in w hours. A man working alone and at a constant rate can do the same job in m hours. The woman works alone at her constant rate for 2 hours; the man then joins her and each works at his/her constant rate until the job is finished. Assume no loss of efficiency and assume that the rates remain constant. Find the length of time in hours after the woman started until the job is finished. Express your answer as a simplified single fraction and express your answer in terms of w and m.

13. Let $5(x+y) = xy$. If x and y are positive integers, find the sum of all possible distinct values of x.

14. In the diagram, $\odot A$ and $\odot B$ intersect in points E and F. $\odot A$ has a radius whose length is 5, and $\odot B$ has a radius whose length is 9. $\overline{AB}$, the segment connecting the centers of the circle, has a length of 12. $EF = \frac{k\sqrt{w}}{p}$ where k, w, and p are positive integers. Find the smallest possible value of $(k+w+p)$.



NO CALCULATORS

NO CALCULATORS

15. Let x and y be positive integers with $x > y$. Find the **ordered pair** (x, y) such that the quotient of x and y and the positive difference of x and y are equal.

16. Let x, y, k and $\sqrt{141+k}$ be positive integers and $k < 321$. Let $S = x + y + k$. Find the sum of all possible distinct values of S such that $x^2 = 141 + k - y^2$.

17. Let a and b be non-zero real numbers. $\left(\frac{1}{4}a^2b^{-3}\right)^{-1}\left(\frac{1}{2}a^{-3}b^2\right)^{-2}\left(2a^3b^{-3}\right)^{-4} = pa^kb^w$ where p is positive and k and w are integers. Find the least value of $(k + w + p)$.

18. Let $A = \{3, 4, 5, 6\}$. From A, three distinct elements are selected at random and used as the lengths of the three sides of a triangle. Find the probability that this triangle is obtuse. Express your answer as a common fraction reduced to lowest terms.

19. The 10-digit number $abc35b62ca$, where $a > b > c$ represent non-zero digits, is divisible by 792. Find the ordered triple (a, b, c)

20. Let $x \in \{111, 114, 118, 123, 129, 136, 144, 153, 163, 174\}$. Find the sum of all distinct x such that $\frac{x^3 + 9x^2 + 17x + 6}{6}$ is an integer.

NO CALCULATORS

2011 SA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

_____ Correct X **5** pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 25
2. 34
3. 51
4. 150
5. 10
6. $(1,-1)$ (Must be this ordered pair.)
7. Never (Must be the whole word.)
8. 76 OR 76_{10} OR 76_{ten}
9. 14
10. 1011243
11. 61 (Degrees optional.)
12. $\frac{(mw+2w)}{m+w}$ OR $\frac{(m+2)w}{m+w}$ (Must be single fraction, equivalent commutations acceptable.)
13. 46
14. 33
15. $(4,2)$ (Must be this ordered pair.)
16. 608
17. 4
18. $\frac{1}{2}$ (Must be this reduced common fraction.)
19. $(8,7,4)$ (Must be this ordered triple.)
20. 686