JUNIOR-SENIOR EIGHT PERSON TEAM COMPETITION
ICTM STATE 2011 DIVISION A

NO CALCULATORS

1. **(Always, Sometimes, or Never true)** For your answer, write <u>*the whole word*</u> **Always, Sometimes,** or **Never**—whichever is correct for the following statement:

   "The converse of a false statement is a true statement."

2. Find the **sum** of all distinct integers in the domain of the real-valued function $f(x)=\sqrt{8x-x^2}$.

3. When $6x^3+4x^2+kx+3$ is divided by $(x-2)$, the remainder is 7. Find the value of k .

4. Let $\|(k,w)\|$ represent the **norm** of the vector represented by (k,w). Find the exact value of $\|(7,9)\|$.

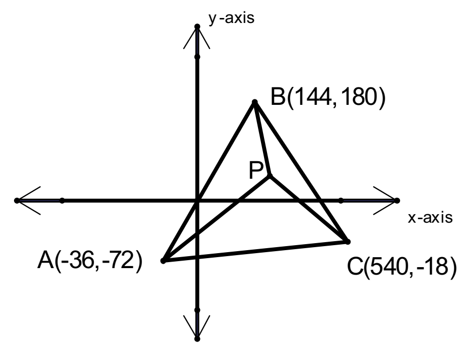
5. An ellipse has an equation of $\frac{16(x-5)^2}{289}+\frac{(y+3)^2}{64}=1$. The area of this ellipse can be expressed in the form $k\pi$. Find the value of k .

6. Bob and Judy are playing a game with four dice whose faces are numbered as follows:

Orange Die	4,	4,	4,	4,	4,	4	
Blue Die	8,	8,	2,	2,	2,	2,	2
Scarlet Die	7,	7,	7,	1,	1,	1	
Gray Die	6,	6,	6,	6,	0,	0	

Bob must select a die first. After Bob selects his die, and after he rolls that die, Judy must select her die. Judy then rolls once. The winner is the one who rolled the higher number. Bob selects the scarlet die and rolls. If Judy applies the poorest possible strategy, find the probability that Judy will win the game. Express your answer as a common fraction reduced to lowest terms.

7. In the diagram with ΔABC and coordinates as shown, the ratio of the area of ΔPAB to the area of ΔPBC to the area of ΔPAC is $2:3:4$. Find the **ordered pair** that represents point P .



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8. Find the reciprocal of the sum of the reciprocals of 2 and 4. Express your answer as an improper fraction reduced to lowest terms.

9. Find the **sum** of all distinct values of k for which the relation S will **not** be a function if $S=\{(|k+2|+3,19),(8,7),(15,1)\}$.

10. If $2^x-7=0$, then $x=\dfrac{\log_4 k}{\log_{64} w}$ where k and w are positive integers. Find the smallest possible value of $(2k+3w)$.

11. Given the four letters: A, B, C, D. $ABCAD$ and $ABABD$ are two examples of a five letter movement. In a five letter movement, one must start with the letter A and end with the letter D. No two consecutive spots in a five letter movement can consist of the same letter. For example, ABBCD would **not** be a five letter movement. Choosing only from the four given letters, find the number of distinct five letter movements.

12. Let b and c be real numbers such that $|b|\le 3$ and $|c|\le 3$, and such that at least one of those two variables (b and c) must represent an integer. For how many different values of x can the two solutions for x of the equation $x^2+bx+c=0$ be equal?

13. $\begin{bmatrix} 2 & -3 \\ a & c \end{bmatrix}\begin{bmatrix} a & 4 \\ c & 2 \end{bmatrix}=\begin{bmatrix} 16 & 2 \\ 20 & 0 \end{bmatrix}$. Find the value of $(a+c)$.

14. The arithmetic mean of four numbers, none of which is a prime, is 21. A prime number between 26 and 30 is added to the list to make five numbers. Let k and w be two **new** prime numbers between 16 and 40 that are added to the list to make a total of seven numbers. The arithmetic mean of these seven numbers is 23. Find the least possible value of $|k-w|$.

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15. Line L passes through the point represented by $(7,-9)$ and is perpendicular to the line passing through points represented by $(21,10)$ and $(7,8)$. Line L can be represented as $\{(7+t,-9+kt)\}$. Find the value of k.

16. **(Always, Sometimes, or Never true)** For your answer, write _the whole word_ **Always, Sometimes,** or **Never**—whichever is correct for the following statement:

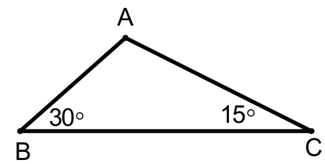
"If x is a real number, then $|x|<7$ is equivalent to $-7<x<7$."

17. Cindy tosses a fairly weighted penny six times. Find the probability that at least four consecutive tosses were heads. Express your answer as a common fraction reduced to lowest terms.

18. Let $i=\sqrt{-1}$, and let k and w represent integers. If the product of the roots for x for the quadratic equation $x^2+5ix=kx+wi$ is $-35i$, find the value of w.

19. One of the angle bisectors of the angles formed by the graphs of $15x-20y+17=0$ and $20x+15y-12=0$ is represented by the equation $kx-y+w=0$ where k and w are integers. Find the value of $(k+w)$.

20. In ΔABC, $BC=12$, $\angle ABC=30°$, and $\angle ACB=15°$. The area of ΔABC can be expressed as $k\sqrt{w}-p$ where k, w, and p are positive integers. Find the smallest possible value of $(k+w+p)$.



2011 SA

School ANSWERS

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. Sometimes (Must be the whole word.)
2. 36
3. -30
4. $\sqrt{130}$ (Must be this exact answer.)
5. 34
6. $\frac{1}{3}$ (Must be this reduced common fraction.)
7. $(172, 52)$ (Must be this ordered pair.)
8. $\frac{4}{3}$ (Must be this reduced improper fraction.)
9. -8
10. 38
11. 20
12. 9
13. -2
14. 14
15. -7
16. Always (Must be the whole word.)
17. $\frac{1}{8}$ (Must be this reduced common fraction.)
18. 35
19. 8
20. 39