1. Let $i=\sqrt{-1}$. If k and w are real numbers such that $\frac{k+wi}{19-7i}=2$, find the value of $(k+w)$.

2. Let k represent a positive integer such that $3<k<1023$. Find the largest possible value of k such that $\sin^2(k°)+\cos^2(k°)-1=0$.

3. **(Always, Sometimes, or Never true)** For your answer, write *the whole word* **Always, Sometimes,** or **Never**—whichever is correct.

   If a function is continuous at $x=c$, then the limit of the functions value does **not** exist as $x\to c$.

4. For the equation $x^2+x+k=0$, the square of the difference between the roots for x is 40 more than the sum of the squares of these roots for x. Find the value of k for which this is true.

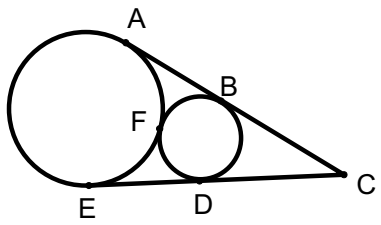
5. Find the value of $\sum_{k=1}^{\infty}\left(\left(\frac{1}{2}\right)^k\right)$.

6. Let $\vec{a}$, $\vec{b}$, $\vec{c}$, and $\vec{d}$ represent vectors such that $\vec{a}=(3,2)$, $\vec{b}=(-4,3)$, and $\vec{c}=(17,6)$. Find the **ordered pair** representing $\vec{d}$ if $\vec{a}-\vec{b}=\vec{c}+\vec{d}$.

7. If $(a+b)^{18}$ is expanded and completely simplified, find the sum of the squares of the numerical coefficients.

8. Find the value of the indicated sum: $\sum_{x=3}^{4}(3x+4^x)$.

9. In the diagram, $\overleftrightarrow{AC}$ and $\overleftrightarrow{EC}$ are common external tangents of the two circles with points of tangency at A, B, D, and E. The circles are tangent at F and have radii of lengths 3 and 8. Then $\sin\angle BCD = \frac{k\sqrt{w}}{f}$ where k, w, and f, are positive integers. Find the smallest possible value of $(k+w+f)$.



10. Let x represent the degree measure of an angle such that $\tan(x)=\sqrt{3}$. If $180<x<270$, find the value of x.

11. Let x represent an integer such that $0\le x\le 360$. Find the sum of all distinct values of x such that $2(\sin(x^\circ)+\cos(x^\circ))<\sqrt{1+\sin(x^\circ)\cos(x^\circ)}$.

12. A parabola has its line of symmetry parallel to the *x*-axis and has its vertex at $(7,2)$. The point $(3,-8)$ lies on the parabola. Find the ***x*-coordinate only** of the focus of this parabola. Express your answer as a **decimal**.

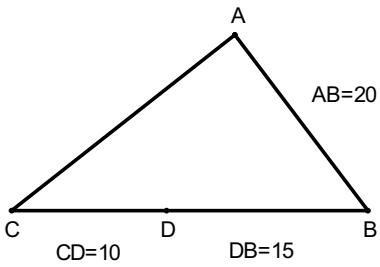
13. In taking a ten problem multiple choice test with 5 choices for each problem, a student randomly guesses on all ten questions. Find the probability that the student guessed at least three correct answers out of the ten. Express your answer as a **decimal** rounded to the nearest ten-thousandth.

14. Let $f(x)=x^7+x^6+x^5+x^4+x^3+x^2+x+1$. If k is a positive integer such that $53<k<115$, find the sum of all distinct k such that the numerical remainder when the polynomial $f(x^k)$ is divided by the polynomial $f(x)$ **must** be 8.

15. Let k represent the degree measure of an angle such that $\sin(33°) = \cos(k°)$. If $270° < k < 360°$, find the value of k ..

16. The sum of the last two terms of a eight term geometric progression of real terms is $\frac{2}{9}$. The sum of the third and fourth terms of this geometric progression is 18. Find the sum of all eight terms of this geometric progression. Express your answer as an improper fraction reduced to lowest terms.

17. In ΔABC, $\angle ABC = 70°$ and $AB = 20$. D lies on $\overline{BC}$ such that $CD = 10$ and $DB = 15$. Let E be a point on $\overline{AB}$ such that $CE + ED$ is as small as possible. Find that value of $CE + ED$. Express your answer as a decimal rounded to the nearest hundredth.



18. Let $C(n,k) = \frac{n!}{k!(n-k)!}$ where n and k represent positive integers. Find the value of n such that $C(n,7) = 330$.

19. When $(2x+3y)^5$ is expanded and completely simplified, the coefficient of one of the terms is 240. Find the **exponent** of x for that term.

20. If x is a real number, find the number of distinct values of x that satisfy the equation $x = 50\cos(x)$.

2011 SA
Pre-Calculus

Name ANSWERS

School _____
(Use full school name – no abbreviations)

_____ **Correct X 2 pts. ea. =** ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 24

2. 1022

3. Never (Must be the whole word.)

4. -20

5. 1

6. $(-10,-7)$ (Must be this ordered pair.)

7. 9,075,135,300 (Commas optional.)

8. 341

9. 165

10. 240 (Degrees optional.)

11. 47475 (Degrees optional.)

12. 0.75 OR .75 (Must be this decimal.)

13. 0.3222 OR .3222 (Must be this decimal.)

14. 672

15. 303 (Degrees optional.)

16. $\frac{1640}{9}$ (Must be this reduced improper fraction.)

17. 37.74 (Must be this decimal.)

18. 11

19. 4

20. 31