

NO CALCULATORS

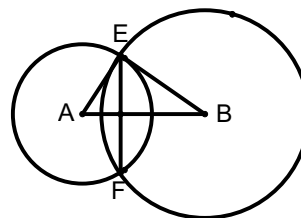
1. A second point is located sixteen units above and two units to the left of (k, w) . If $(k + w) = 24$, find the sum of the coordinates of the second point.
2. Find the absolute value of the difference between the geometric mean of 24 and 54 and the arithmetic mean of 24 and 54.
3. The solution set for the inequality $\frac{2}{x} + 8 > 5 - \frac{3}{x}$ is $\{x : x > k \text{ or } x < w\}$. Find the value of $(k + w)$. Answer as a reduced improper fraction.
4. $KATE$ is a parallelogram with diagonals that intersect at point I . $KA = 8x - 18$, $KI = 4x - 6$, $IA = 15 - 2x$, and $IT = 2x + 3$. Find the length of the longer diagonal of $KATE$.
5. Given rectangle $RECT$ with $RE = 8$ and with 17 as the length of a diagonal of this rectangle. If a point is randomly chosen on the sides of the rectangle, find the probability that it lies on one of the longer sides of the rectangle. Express your answer as a common fraction reduced to lowest terms.
6. In a circle whose equation is $x^2 + y^2 + 10x - 14y - 26 = 0$, $(-11, 15)$ is one endpoint of a diameter of the circle. Find the coordinates of the other endpoint of this diameter.
7. In $\triangle ABC$, the vertices have coordinates $A(2, 1)$, $B(16, 7)$, and $C(8, 12)$. Which one point does NOT lie on a side of the triangle with negative slope? Answer with the **capital letter** A , B , or C .
8. When expressed in base fifty-six, $N!$ terminates in a block of exactly 11 zeroes. Compute the largest positive integer N with this property. Express your answer for N in base ten notation.

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9. Two chords intersect inside a circle. The first chord is divided into two segments of length 42 and 18. The second chord is divided into two segments of length 35 and k . Find the value of k . Express your answer as a reduced improper fraction.
10. A woman working alone and at a constant rate can do a job in w hours. A man working alone and at a constant rate can do the same job in m hours. The woman works alone at her constant rate for 2 hours; the man then joins her and each works at his/her constant rate until the job is finished. Assume no loss of efficiency and assume that the rates remain constant. Find the length of time in hours after the woman started until the job is finished. Express your answer as a simplified single fraction and express your answer in terms of w and m .
11. Points A , B , C , and D lie in plane m . No three of the four points are collinear. From point P outside the plane, \overline{PA} is drawn such that $\angle PAB = (3x + 4y - 49)^\circ$, $\angle PAC = (5x - 2y + 75)^\circ$, and $\angle PAD = (4x - y - 30 + k)^\circ$. Find the value of k so that $\overline{PA} \perp m$.
12. $4\sqrt{18} - 2\sqrt{27} - \frac{5}{2}\sqrt{2} + \frac{2}{3}\sqrt{3} = \frac{a\sqrt{b} - c\sqrt{d}}{e}$ where a , b , c , d , and e are positive integers. Find the least possible sum $(a + b + c + d + e)$.

13. In the diagram, $\odot A$ and $\odot B$ intersect in points E and F . $\odot A$ has a radius whose length is 5, and $\odot B$ has a radius whose length is 9. \overline{AB} , the segment connecting the centers of the circle, has a length of 12. $EF = \frac{k\sqrt{w}}{p}$ where k , w , and p are positive integers. Find the smallest possible value of $(k + w + p)$.



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14. In a class of 20 students, the 12 boys averaged 6'4" (6 feet, 4 inches) in height. If the entire class averaged exactly 6' in height, how tall was the average height of a girl? Answer in the form $k'w''$.
15. Find the value of $(168542)^2 - (168539)^2$.
16. Let x , y , k and $\sqrt{141+k}$ be positive integers and $k < 321$. Let $S = x + y + k$. Find the sum of all possible distinct values of S such that $x^2 = 141 + k - y^2$.
17. In a certain circle, a 56° central angle intercepts a minor arc of length $\frac{14}{3}\pi$. Find the length of the radius of the circle.
18. Let x , y , and z be positive integers such that $x < y < z$ and $x + y + z = 12$. Find the largest possible value of $(y + \frac{1}{z})^2$. Express your answer as an improper fraction reduced to lowest terms.
19. The local Humane Society had at least one dog and one cat in residence on Monday. On Tuesday, the number of cats tripled while the number of dogs stayed the same. On Wednesday, 9 cats were adopted out (left the Society), but one dog came into the Humane Society. On Thursday, the number of cats stayed the same, but the number of dogs doubled. On Friday, 2 dogs and 2 cats were adopted out, leaving 35 animals to spend the weekend at the Humane Society. How many distinct ordered pairs of the form (number of dogs, number of cats) were possible originally on Monday?
20. Let $x \in \{111, 114, 118, 123, 129, 136, 144, 153, 163, 174\}$. Find the sum of all distinct x such that $\frac{x^3 + 9x^2 + 17x + 6}{6}$ is an integer.

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2011 SAA

School _____ **ANSWERS** _____

Fr/So 8 Person

(Use full school name – no abbreviations)

_____ Correct X **5** pts. ea. =

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. _____ 38 _____

11. _____ 93 _____

2. _____ 3 _____

12. _____ 100 _____

3. _____ $-\frac{5}{3}$ OR $\frac{-5}{3}$ OR $\frac{5}{-3}$ (Must be this reduced improper fraction.) _____

13. _____ 33 _____

4. _____ 24 _____

14. _____ $5'6''$ (Must be in this format.) _____

5. _____ $\frac{15}{23}$ (Must be this reduced common fraction.) _____

15. _____ 1011243 _____

6. _____ $(1, -1)$ (Must be this ordered pair.) _____

16. _____ 608 _____

7. _____ A (Must be this capital letter.) _____

17. _____ 15 _____

8. _____ 76 OR 76_{10} OR 76_{ten} _____

18. _____ $\frac{961}{36}$ (Must be this reduced improper fraction.) _____

9. _____ $\frac{108}{5}$ (Must be this reduced improper fraction.) _____

19. _____ 6 (“Ordered pairs optional.”) _____

10. _____ $\frac{mw + 2w}{m + w}$ OR $\frac{(m + 2)w}{m + w}$ (Must be this single fraction, equivalent commutations acceptable.) _____

20. _____ 686 _____