NO CALCULATORS

1. **(Yes or No)** Write the *whole word* to answer whether or not the conclusion drawn from the two given statements is valid?

   (1) If Tom is handsome, then Kay is beautiful.
   (2) Kay is beautiful.

   Conclusion: Tom is handsome.

2. Find the **sum** of all distinct integers **not** in the domain of the real-valued function: $f(x)=\sqrt{x^2+10x+21}$.

3. Find the exact value of $\left(\log_3 2\right)\left(\log_4 3\right)\left(\log_5 4\right)\cdots\left(\log_{8191} 8190\right)\left(\log_{8192} 8191\right)$. Express your answer as a fraction.

4. Let $\|(k,w)\|$ represent the **norm** of the vector represented by (k,w). Find the exact simplified value of $\|(4,8)\|$.

5. An ellipse has an equation of $\frac{16(x-5)^2}{289}+\frac{(y+3)^2}{64}=1$. The area of this ellipse can be expressed in the form $k\pi$. Find the value of k.

6. Find the value of k if $(3x+1)$ is a factor of $54x^3-9kx^2-6kx+12$.

7. **(Always, Sometimes, or Never true)** For your answer, write *the whole word* **Always, Sometimes, or Never**—whichever is correct for the following statement:

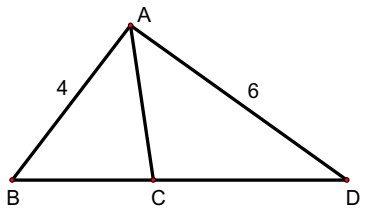
   "If x represents an odd positive integer, then x^3-x is an integral multiple of 24."

NO CALCULATORS

8. If $2^x - 7 = 0$, then $x = \dfrac{\log_4 k}{\log_{64} w}$ where k and w are positive integers. Find the smallest possible value of $(2k + 3w)$.

9. Find the **product** of all distinct values of k for which the relation S will **not** be a function if $S = \{(|k+2|+3, 19), (8,7), (15,1)\}$.

10. In ΔABD, $\angle BAC \cong \angle DAC$, points B, C, and D are collinear, $AB = 4$, and $AD = 6$. Let $BC = k$, where k is a positive integer. Find the sum of all possible **positive** values of $\cos(\angle ABC)$. Express your answer as a common fraction reduced to lowest terms.



11. When $6x^3 + kx^2 + 4x + 3$ is divided by $(2x+1)$, the remainder is -1. Find the value of k.

12. Let b and c be real numbers such that $|b| \le 3$ and $|c| \le 3$, and such that at least one of those two variables (b and c) must represent an integer. For how many different values of x can the two solutions for x of the equation $x^2 + bx + c = 0$ be equal?

13. Let $B = \{7, 12, x, 161, 201\}$ where the members of the set are written in ascending order. If the following three values: 12, the median, and the arithmetic mean, taken in that order, form an arithmetic sequence, find the value of x.

NO CALCULATORS

14. Given the four letters: A, B, C, D. $ABCAD$ and $ABABD$ are two examples of a five letter movement. In a five letter movement, one must start with the letter A and end with the letter D. No two consecutive spots in a five letter movement can consist of the same letter. For example, ABBCD would **not** be a five letter movement. Choosing only from the four given letters, find the number of distinct five letter movements.

15. Line L passes through the point represented by $(8,-12)$ and is parallel to the line passing through points represented by $(4,-15)$ and $(15,-23)$. Line L can be represented as $\{(8+t,-12+kt)\}$ Find the value of k. Express your answer as a simplified fraction.

16. $\begin{bmatrix} 2 & -3 \\ a & c \end{bmatrix}\begin{bmatrix} a & 4 \\ c & 2 \end{bmatrix} = \begin{bmatrix} 14 & 2 \\ 17 & -4 \end{bmatrix}$. Find the value of $(a+c)$.

17. Cindy tosses a fairly weighted penny six times. Find the probability that at least three consecutive tosses were heads. Express your answer as a common fraction reduced to lowest terms.

18. Let $i=\sqrt{-1}$, and let k and w represent integers. If the sum of the roots for x for the quadratic equation $x^2+wix+k=2ix$ is $-i$, find the value of w.

19. One of the angle bisectors of the angles formed by the graphs of $15x-20y+17=0$ and $20x+15y-12=0$ is represented by the equation $kx-y+w=0$ where k and w are integers. Find the value of $(k+w)$.

20. Let $0\le a\le b\le c\le d\le 4$ and $a+b+c+d=4$. If $d\le 2a$, find the minimum possible value of the product $abcd$. Express your answer as a common fraction reduced to lowest terms.

2011 SAA

Jr/Sr 8 Person

School ANSWERS

(Use full school name – no abbreviations)

_____ Correct X **5** pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. No (Must be the whole word.)
2. -15
3. $\frac{1}{13}$ (Must be this reduced common fraction.)
4. $4\sqrt{5}$ (Must be this exact simplified radical.)
5. 34
6. -10
7. Always (Must be the whole word.)
8. 38
9. 2940
10. $\frac{35}{48}$ (Must be this reduced common fraction.)
11. -5
12. 9
13. 49
14. 20
15. $-\frac{8}{11}$ OR $\frac{-8}{11}$ OR $\frac{8}{-11}$ (Must be this reduced common fraction.)
16. -3
17. $\frac{5}{16}$ (Must be this reduced common fraction.)
18. 3
19. 8
20. $\frac{64}{81}$ (Must be this reduced common fraction.)