

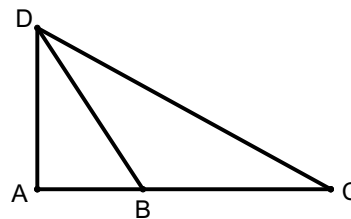
1. Let  $i = \sqrt{-1}$ . If  $k$  and  $w$  are real numbers such that  $\frac{k + wi}{10 - 24i} = 5$ , find the value of  $(k + w)$ .

2. Let  $k$  be a real number. Find the value of  $\sin^2(3567894k) + \cos^2(3567894k)$ .

3. Find the value of the indicated sum:  $\sum_{k=1}^4 (2k + 3)$ .

4. Find the sum of the terms of an infinite geometric sequence whose first four terms are  $2, -1, \frac{1}{2}, -\frac{1}{4}$ . Express your answer as an improper fraction reduced to lowest terms.

5. In the diagram, points  $A, B$ , and  $C$  are collinear.  
 $\angle DBA = 17.13^\circ$  and  $\angle DCA = 11.87^\circ$ . If  $DA = 967.3$ , find  $BC$ .  
Round your answer to the nearest **integer** and express your answer as that **integer**.

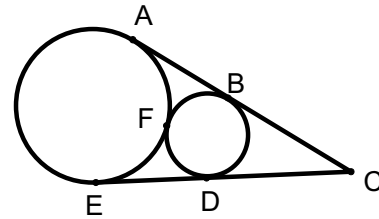


6. Let  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$ , and  $\vec{d}$  represent vectors such that  $\vec{a} = (3, 2)$ ,  $\vec{b} = (-8, 13)$ , and  $\vec{c} = (19, -86)$ . Find the **ordered pair** representing  $\vec{d}$  if  $\vec{a} - \vec{b} = \vec{c} + \vec{d}$ .

7. When the sum of the first  $k$  terms of the series  $1^2 + 2^2 + 3^2 + \dots + n^2 + \dots$  is subtracted from the sum of the first  $k$  terms of the series  $1(2) + 2(3) + 3(4) + \dots + n(n+1) + \dots$ , the result is 528. Find the value of  $k$ .

8. For the equation  $x^2 + 11x + k = 0$ , the square of the difference between the roots for  $x$  is 120 more than the sum of the squares of these roots for  $x$ . Find the value of  $k$  for which this is true.

9. In the diagram,  $\overline{AC}$  and  $\overline{EC}$  are common external tangents of the two circles with points of tangency at  $A$ ,  $B$ ,  $D$ , and  $E$ . The circles are tangent at  $F$  and have radii of lengths 3 and 8. Then  $\sin \angle BCD = \frac{k\sqrt{w}}{f}$  where  $k$ ,  $w$ , and  $f$ , are positive integers. Find the smallest possible value of  $(k + w + f)$ .



10. Let  $x$  represent the degree measure of an angle such that  $\cot(x) = \sqrt{3}$ . If  $180^\circ < x < 270^\circ$ , find the value of  $x$ .
11. Urn A contains five marbles—three of which are orange and two of which are blue. Urn B contains four marbles—three of which are orange and one of which is blue. One of the urns is selected at random, and a marble then is selected at random from that urn. If the marble selected was orange, find the probability that the marble came from Urn B. Express your answer as a common fraction reduced to lowest terms.
12. A parabola has its line of symmetry parallel to the  $x$ -axis and has its vertex at  $(7, 2)$ . The point  $(3, -8)$  lies on the parabola. Find the  **$x$ -coordinate only** of the focus of this parabola. Express your answer as a **decimal**.
13. Let  $\overline{AB}$ ,  $\overline{CD}$ , and  $\overline{EF}$  be three parallel chords that are non-diameters of a circle on the same side of the center. Let  $\overline{GJ}$  be tangent to the circle at  $H$  such that  $\overline{GJ} \parallel \overline{AB}$ . The distance between  $\overline{AB}$  and  $\overline{CD}$  is equal to the distance between  $\overline{CD}$  and  $\overline{EF}$  and is also equal to the distance between  $\overline{EF}$  and  $\overline{GJ}$ . If  $AB = 24$ , then  $CD$  **must** be greater than  $k$ . Find the largest possible simplified exact value of  $k$ .

14. A geometric sequence consists of 10 terms. The sum of the 10 terms is 2343.74976, and the sum of the reciprocals of each of the 10 terms is 1302.0832. Find the product of the 10 terms of the original sequence. Express your answer as an exact decimal.
15. Let  $k$  represent a positive integral degree measure such that  $\frac{\sin(57^\circ)}{\cos(k^\circ)} = 1$ . If  $0^\circ < k < 90^\circ$ , find the value of  $k$ .
16. The sum of the last two terms of an eight term geometric progression of real terms is  $\frac{2}{9}$ . The sum of the third and fourth terms of this geometric progression is 18. Find the sum of all eight terms of this geometric progression. Express your answer as an improper fraction reduced to lowest terms.
17. In rhombus  $ABCD$ ,  $\angle DAB = 52^\circ$ . A circle passes through vertices  $A$ ,  $B$ , and  $D$ , and intersects diagonal  $\overline{AC}$  at  $E$ .  $CE = 12$ . Find the length of the arc of the circle from  $A$  to  $D$  to  $B$ . Express your answer as a decimal rounded to the nearest hundredth.
18. Let  $C(n, k) = \frac{n!}{k!(n-k)!}$  where  $n$  and  $k$  represent positive integers. Find the value of  $n$  such that  $C(n, 5) = 792$ .
19. When  $(2x + 3y)^5$  is expanded and completely simplified, the coefficient of one of the terms is 1080. Find the **exponent** of  $x$  for that term.
20. By substituting 1, 2, 3, 4, 5, and 6 for  $x$  into a polynomial expression with integral coefficients in  $x$ , the values are respectively  $-3$ ,  $10$ ,  $49$ ,  $120$ ,  $229$ , and  $382$ . If  $P(x)$  is the polynomial expression with integer coefficients of lowest degree satisfying the given conditions, find  $P(43)$ .

# 2011 SAA

Name ANSWERS

## Pre-Calculus

School \_\_\_\_\_

(Use full school name – no abbreviations)

\_\_\_\_\_ Correct X 2 pts. ea. = 

**Note:** All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1.                     -70                    

11.                      $\frac{5}{9}$                      (Must be this reduced common fraction.)

2.                                     1                                    

12.                     0.75 OR .75                     (Must be this decimal.)

3.                                     32                                    

13.                      $8\sqrt{6}$                      (Must be this exact simplified radical.)

4.                      $\frac{4}{3}$                      (Must be this reduced improper fraction.)

14.                     18.89568                     (Must be this exact decimal.)

5.                     1464                     (Must be this integer.)

15.                                     33                                     (Degrees optional.)

6.                     (-8, 75)                     (Must be this ordered pair.)

16.                      $\frac{1640}{9}$                      (Must be this reduced improper fraction.)

7.                                     32                                    

17.                     39.46                     (Must be this exact decimal.)

8.                                    -60                                   

18.                                     12                                    

9.                                     165                                    

19.                                     2                                    

10.                     210                     (Degrees optional.)

20.                     91809